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SESSION 3: NUMBERS & NUMBER PATTERNS

KEY CONCEPTS:

- Understand that real numbers can be irrational or rational

- Establish between which two integers a given   simple surd lies.

- Round real numbers to an appropriate degree of accuracy. irrational or rational

- Establish between which two integers a given simple surd lies.

- Round real numbers to an appropriate degree of accuracy.

- Investigate number patterns leading to those where there is constant difference between consecutive terms, and the general term is therefore linear.

X-PLANATION

The Real Number System
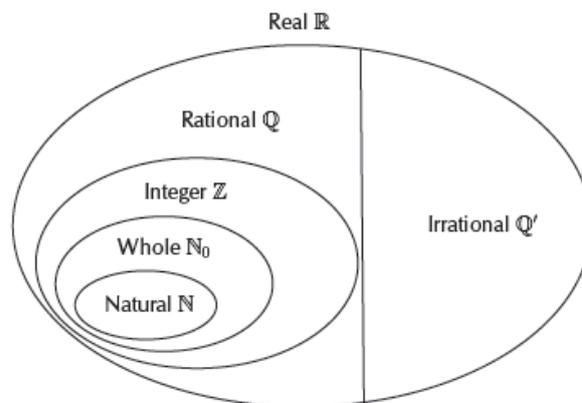
We use the following definitions:

- $\mathbb{N}$: natural numbers are $\{1;\ 2;\ 3;\ \ldots\}$

- $\mathbb{N}_0$: whole numbers are $\{0;\ 1;\ 2;\ 3;\ \ldots\}$

- $\mathbb{Z}$: integers are $\{\ldots;\ -3;\ -2;\ -1;\ 0;\ 1;\ 2;\ 3;\ \ldots\}$

- Q': Irrationals are: Non-terminating, non-recurring decimals
  e.g. π; √2

Numbers

Converting recurring decimals to fractions

Questions:

Write $0,\dot{3}$ in the form $\frac{a}{b}$ (where a and b are integers)

Solution:

Step 1: **Define an equation**

Let $x = 0,33333...$

Step 2: **Multiply by 10 on both sides**

$10x = 3,33333...$

Step 3: **Subtract the first equation from the second equation**

$9x = 3$

Step 4: **Simplify**

$x = \frac{3}{9} = \frac{1}{3}$

Questions:

Write $5,4\overset{...}{32}$ as a rational fraction.

Solution:

Step 1: **Define an equation**

$x = 5,432432432...$

Step 2: **Multiply by 1000 on both sides**

$1000x = 5\ 432,432432432...$

Step 3: **Subtract the first equation from the second equation**

$999x = 5\ 427$

Step 4: **Simplify**

$x = \dfrac{5\ 427}{999} = \dfrac{201}{37} = 5\dfrac{16}{37}$

Estimating Surds

Question:

Find the two consecutive integers such that $\sqrt{26}$ lies between them.
(Remember that consecutive integers are two integers that follow one another on the number line, for example, 5 and 6 or 8 and 9).

Solution:

Step 1: **Use perfect squares to estimate the lower integer**

$5^2 = 25$. Therefore $5 < \sqrt{26}$.

Step 2: **Use perfect squares to estimate the upper integer.**

$6^2 = 36$. Therefore $\sqrt{26} < 6$.

Step 3: **Write the final answer**

$5 < \sqrt{26} < 6$.

Question:

Find the two consecutive integers such that $\sqrt[3]{49}$ lies between them.

Solution:

Step 1: Use perfect cubes to estimate the lower integer

$3^3 = 27$, therefore $3 < \sqrt[3]{49} < 4$

Step 2: Use perfect cubes to estimate the upper integer

$4^3 = 64$, therefore $\sqrt[3]{49} < 4$

Step 3: Write the answer

$3 < \sqrt[3]{49} < 4$

Step 4: Check the answer by cubing all terms in the inequality and then simplify

$27 < 49 < 64$. This is true, so $\sqrt[3]{49}$ lies between 3 and 4

Number Patterns

Investigation

You and 3 friends are studying for Maths and are sitting together at a square table. A few minutes later 2 other friends arrive to you move another table next to yours. Now 6 people can sit at the table. Another 2 friends also join your group, so you take a third table and add it to the existing tables. Now 8 people can sit together as shown below.

1.   Find the expression for the number of people seated at *n* tables.

2.   Use the general formula to determine how many people can sit around 12 tables.

3.   How many tables are needed to seat 20 people?
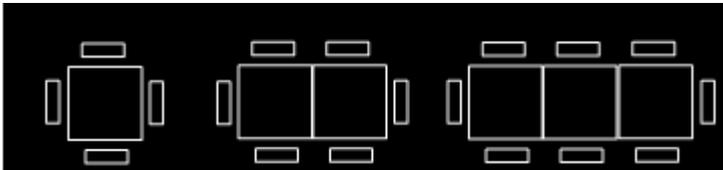
Two more people can be seated for each table added.

Solutions:

Step 1: Make a table to see the pattern

Number of Tables, n	Number of people seated	Pattern
1	$4 = 4$	$= 4 + 2(0)$
2	$4 + 2 = 6$	$= 4 + 2(1)$
3	$4 + 2 + 2 = 8$	$= 4 + 2(2)$
4	$4 + 2 + 2 + 2 = 10$	$= 4 + 2(3)$
$\vdots$	$\vdots$	$\vdots$
n	$4 + 2 + 2 + 2 + \cdots + 2$	$= 4 + 2(n - 1)$

Step 2: Describe the pattern

The number of people seated at n tables is

$T_n = 4 + 2(n - 1)$

**Step 3: Calculate the 12th term, in other words,
        find T_n if n = 12**

$T_{12} = 4 + 2(12 - 1)$
$= 4 + 2(11)$
$= 4 + 22$
$= 26$

Therefore 26 people can be seated at 12 tables.

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Step 4: **Calculate the number of tables needed to seat 20 people, in other words, find n if $T_n = 20$.**

$T_n = 4 + 2(n - 1)$

$20 = 4 + 2(n - 1)$

$20 - 4 = 2(n - 1)$

$\dfrac{16}{2} = n - 1$

$8 + 1 = n$

$n = 9$

Therefore 9 tables are needed to seat 20 people.

X-AMPLE QUESTIONS:

Question 1:
If a is an integer, b is an integer and c is irrational, which of the following are rational numbers?

(a) $\dfrac{-b}{a}$

(b) $c \div c$

(c) $\dfrac{a}{c}$

(d) $\dfrac{1}{c}$

Question 2
Write each decimal as a simple fraction.

(a)   0,12

(b)   0,006

(c)   1,59

(d)   12, 277

Question 3
Express 0,78 as a fraction a/b where *a*, *b* ε Z (Show all working).

Question 4
Use your calculator (where necessary) and write the following numbers to 5 decimal places. State whether the numbers are irrational or rational.

(a) $\sqrt{8}$
(b) $\sqrt{768}$
(c) $\sqrt{0,49}$
(d) $\sqrt{0,0016}$
(e) $\sqrt{0,25}$

Question 5
Determine between which two consecutive integers the following irrational numbers lie, without using a calculator.

(a) $\sqrt{5}$

(b) $\sqrt[3]{5}$

Question 6
Write down the next three terms in each of the following sequences:

(a)   5; 15; 25; ...

(b)   -8; -3; 2 : ' ...

(c)   30; 27; 24; ...

Question 7
The general term is given for each sequence   below. Calculate the missing terms.

(a)   0; 3; ...; 15; 24 $T_n = n^2 - 1$

(b)   3; 2; 1; 0; ...; -2 $T_n = -n + 4$

(c)   -11; ...; -7; ...; -3 $T_n = -13 + 2n$

Question 8
Find the general formula for the following sequences   and then find T_{10}, T_{50} and T_{100}
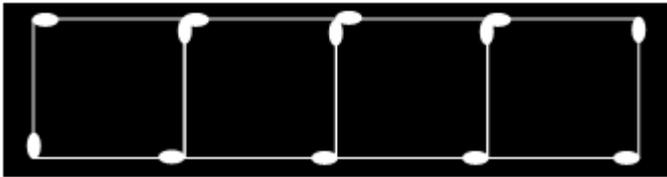
(a)   2; 5; 8; 11; 14; ...
(b)   0; 4; 8; 12; 15; ...

Question 9

The seating of a sports stadium is arranged so that the first row has 15 seats, the second row has 19 seats, the third row has 23 seats and so on.
Calculate how many seats are in the twenty-fifth row.

Question 10
A single square is made from 4 matchsticks. Two squares in a row need 7 matchsticks and three squares in a row need 10 matchsticks.

For this sequence determine:

(a)   the first term;

(b)   the common difference;

(c)   the general formula;

(d)   how many matchsticks there are in a row of twenty-five squares.

Question 11
You would like to start saving some money, but because you have never tried to save money before, you decided to start slowly. At the end of the first week you deposit R5 in your bank account. then at the end of the second week you deposit R10 and at the end of the third week, R15. After how many weeks will you deposit R50 into your bank account?