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SESSION 14: MEASUREMENT

KEY CONCEPTS:

- Surface Area of right prisms, cylinders, spheres, right pyramids and right cones
- Volume of right prisms, cylinders, spheres, right pyramids and right cones
- the effect on volume and surface area when multiplying any dimension by a constant factor *k*.

TERMINOLOGY

Area: the two dimensional space inside the boundary of a flat object. It is measured in square units.

Surface area: the total area of the exposed or outer surfaces of a prism

A net: the unfolded "plan" of a solid.

Volume: the three dimensional space occupied by an object, or the contents of an object. It is measured in cubic units.

A right prism a geometric solid that has a polygon as its base and vertical sides perpendicular to the base. The base and top surface are the same shape and size. It is called a "right" prism because the angles between the base and sides are right angles.

A pyramid: a geometric solid that has a polygon as its base and sides that converge at a point called the apex. The sides are not perpendicular to the base.

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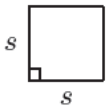
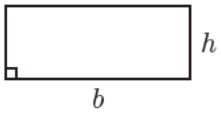
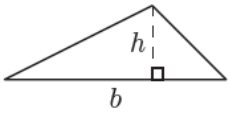
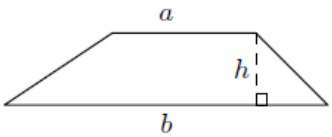
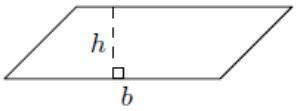
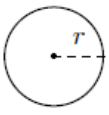
X-PLANATION

Summary: Area of shapes

Name	Shape	Formula
Square		Area $= s^2$
Rectangle		Area $= b \times h$
Triangle		Area $= \frac{1}{2}b \times h$
Trapezium		Area $= \frac{1}{2}(a+b) \times h$
Parallelogram		Area $= b \times h$
Circle		Area $= \pi r^2$ (Circumference $= 2\pi r$)

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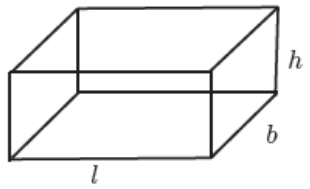
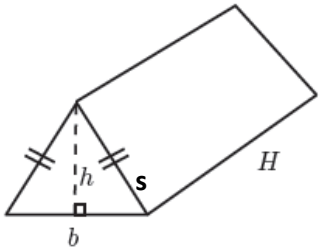
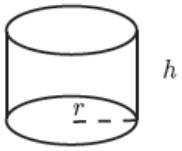
Summary: Surface area of right prisms and cylinders

Rectangular prism		Surface area = 2(l x b + l x h + b x h)
Triangular prism		Surface area = $2(\frac{1}{2}$ x b x h) + b x H + 2(s x H)
Cylinder		Surface area = 2 x π x r^2 +2 x π x r x H

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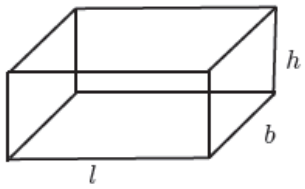
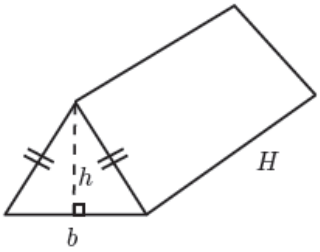
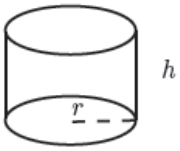
Summary: Volume of right prisms and cylinders

Rectangular prism		Volume = area of base × height = area of rectangle × height $= l \times b \times h$
Triangular prism		Volume = area of base × height = area of triangle × height $= \left(\frac{1}{2}b \times h\right) \times H$
Cylinder		Volume = area of base × height = area of circle × height $= \pi r^2 \times h$

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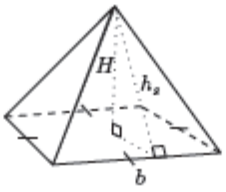
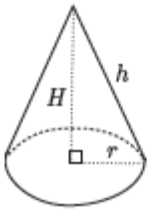
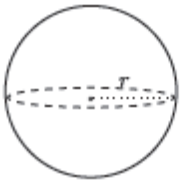
Summary: Surface area of right pyramids, right cones and spheres

Square pyramid		Surface area = area of base + area of triangular sides $= b^2 + 4\left(\frac{1}{2}bh_s\right)$ $= b(b + 2h_s)$
Triangular pyramid		Surface area = area of base + area of triangular sides $= \left(\frac{1}{2}b \times h_b\right) + 3\left(\frac{1}{2}b \times h_s\right)$ $= \frac{1}{2}b(h_b + 3h_s)$
Right cone		Surface area = area of base + area of walls $= \pi r^2 + \frac{1}{2} \times 2\pi rh$ $= \pi r(r + h)$
Sphere		Surface area = $4\pi r^2$

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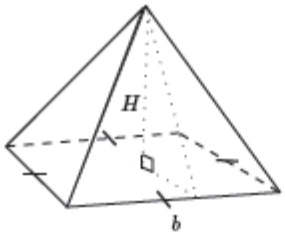
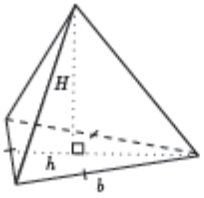
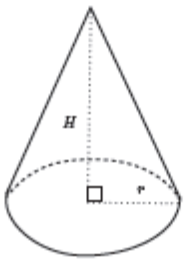
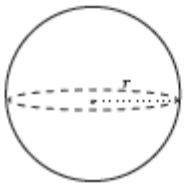
Summary: Volume of right pyramids, right cones and spheres

Square pyramid		Volume $= \frac{1}{3} \times$ area of base $\times$ height of pyramid $= \frac{1}{3} \times b^2 \times H$
Triangular pyramid		Volume $= \frac{1}{3} \times$ area of base $\times$ height of pyramid $= \frac{1}{3} \times \frac{1}{2}bh \times H$
Right cone		Volume $= \frac{1}{3} \times$ area of base $\times$ height of cone $= \frac{1}{3} \times \pi r^2 \times H$
Sphere		Volume $= \frac{4}{3}\pi r^3$

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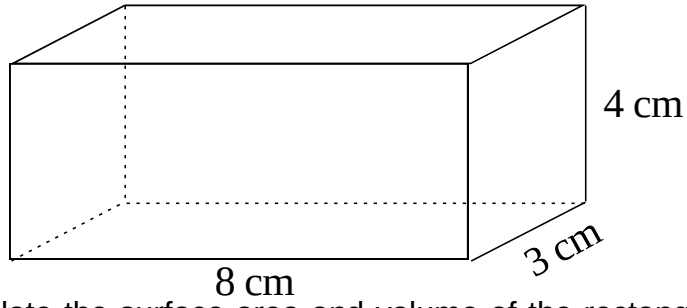
X-AMPLE QUESTIONS:

Question 1:

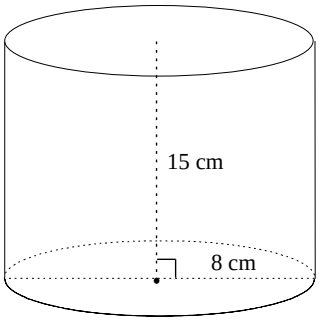


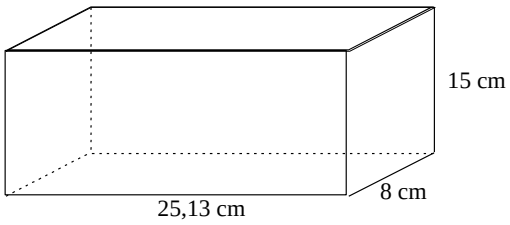
(a) Calculate the surface area and volume of the rectangular prism if it is closed on all sides (4)

(b) Calculate the surface area if the prism is open on top. (2)

Question 2:

A company manufactures dairy products. It has recently created two plastic flavoured milk containers: one is a cylindrical container and the other is a rectangular box. The dimensions are indicated on the diagrams provided below.





(a) Show that the volume of milk in both of the containers is 3016 cm^3 (rounded off to the nearest whole number). (4)

(b) Calculate the surface area of each container (the plastic material). (4)

(c) Although both containers contain the same amount of milk, the company needs to determine which container will be the cheapest to produce. Determine which container will be the cheapest to produce? (2)

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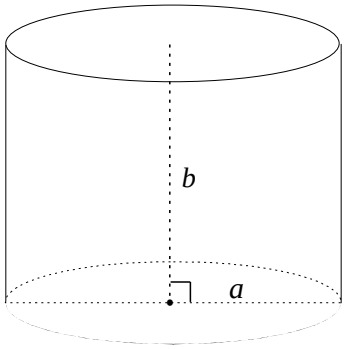
Question 3:
A rectangular prism has a length of 5cm, a breadth of 3cm and a height of 10 cm. The length of each side is increased by a factor of 2,5.

a.) By what factor does the surface area of the prism increase?
b.) By what factor does the volume of the prism increase?

Question 4:

Consider the following cylinder.



(a) Determine the volume in terms of *a* and *b*. (1)

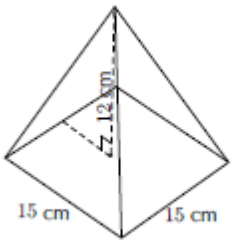
(b) Determine the surface area in terms of *a* and *b*. (1)

(c) If you want to double the volume but keep the radius the same, by what scale factor will the height increase? (3)

(d) If the radius is doubled but the height stays the same, by what number will the area of the base of the cylinder increase? (3)

(e) If the radius is doubled but the height stays the same, by what number will the area of the side surface of the cylinder increase? (3)

Question 5:
Calculate the surface area and volume of the right pyramid with a square base of side length equal to 15 cm and a height of 12 cm



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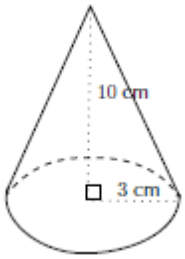
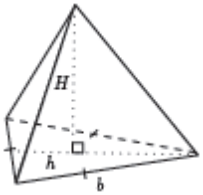
Question 6:
A right cone with a radius of 3cm and a height of 10 cm has the same volume as a right pyramid with an equilateral triangular base. The side length of the base is 5 cm.

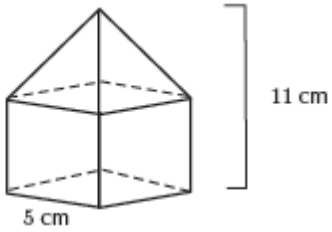




a.) Find the height of the triangular prism
b.) Which solid has the greater surface area?

Question 7:
Calculate the surface area of a dumbbell that consists of a cylinder with a radius of 2cm and length 12cm and two identical spheres which have a radius of 4cm

Question 8:
The solid below is made up of a cube and a square pyramid. Find it's volume and surface area (correct to 1 decimal place):



Question 9:
The height of a cylinder is 10 cm, and the radius of the circular base is 2 cm. A hemisphere is attached to one end of the cylinder and a cone of height 2 cm to the other end. Calculate the volume and surface area of the solid, correct to the nearest cm^3 and cm^2, respectively.