

## SESSION 3: SOLVING QUADRATIC EQUATIONS

### Key Concepts

When you are required to solve a quadratic equation, always get the equation into its standard form  $ax^2 + bx + c = 0$ . There are now three ways to solve a quadratic equation:

- By factorising the trinomial
- By completing the square
- By using the quadratic formula

### X-Planation

#### FACTORISING

**Example:**  $x^2 - x - 12 = 0$   
 $(x + 3)(x - 4) = 0$   
 $x = -3$  or  $x = 4$

#### COMPLETING THE SQUARE

The rule for completing the square is to add in and subtract:

$$\left(\frac{1}{2} \times (\text{the coefficient of } x)\right)^2$$

#### Example

$$x^2 - 2x - 1 = 0$$

$$x^2 - 2x - 1 + 2 = 0 + 2$$

$$\therefore x^2 - 2x + 1 = 2$$

$$\therefore (x - 1)^2 = 2 \text{ (Take the square root of both sides)}$$

$$\therefore (x - 1) = -\sqrt{2} \text{ or } (x - 1) = +\sqrt{2}$$

$$\therefore x = 1 - \sqrt{2} \text{ or } x = 1 + \sqrt{2}$$

Using this method you can write the quadratic equation in the form  $a(x + p)^2 + q$   
 $(x - 1)^2 - 2 = 0$

#### QUADRATIC FORMULA

The formula is:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  where  $a \neq 0$

Make sure that you read the question to find out whether the answers must be left in surd form or in round off decimal form.

## X-ample Questions

### Question 1

Write the following expressions in the form  $a(x+p)^2 + q$  by completing the square:

(a)  $x^2 - 8x + 10$  (3)

(b)  $2x^2 - 7x + 6$  (5)

### Question 2

Solve for  $x$  by completing the square for:

$$2x^2 - 8x - 4 = 0 \quad (4)$$

### Question 3

Solve the following equations:

(a)  $15x^2 - 13x + 2 = 0$  (4)

(b)  $2x(x - 4) = 8$  (Give answer in simplest surd form) (4)

(c)  $x - 5 = \frac{2}{x}$  (rounded off to two decimal digits) (5)

### Question 4

Consider the expression:  $\frac{\sqrt{1-x}}{2x-3}$

Determine the value(s) of  $x$  for which the expression is:

(a) undefined (2)

(b) equal to 0 (1)

(c) real (2)

(d) non-real (2)

## X-ercises

### Question 1

Solve for  $x$ :

(a)  $x - 6(x + 1) = 0$  (2)

(b)  $x^2 - 5x - 6 = 0$  (2)

(c)  $5x(x - 3) = 2$  (rounded off to two decimal places) (5)

### Question 2

(a) For which values of  $x$  will the expression  $\sqrt{\frac{4-x}{4}}$  be real? (3)

(b) If  $P = \sqrt{\frac{a^2 - b^2}{a + b}}$  where  $b > a$ , show that  $P$  is non-real. (3)