

## SESSION 6: ANALYTICAL GEOMETRY

### Key Concepts

In this session we will focus on summarising what you need to know about:

- The distance formula
- The midpoint formula
- The gradient of a line segment joining two points
- Parallel lines
- Perpendicular lines

### X-planation

If AB is the line segment joining the points  $A(x_A; y_A)$  and  $B(x_B; y_B)$ , then the following formulas apply to line segment AB.

#### The Distance Formula

$$AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$$

or  $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

#### The Midpoint Formula

$$M\left(\frac{x_A + x_B}{2}; \frac{y_A + y_B}{2}\right) \text{ where M is the midpoint of AB.}$$

The Gradient of a line segment joining two points

$$\text{Gradient of AB} = \frac{y_B - y_A}{x_B - x_A}$$

#### Parallel lines

Parallel lines have equal gradients. If  $AB \parallel CD$  then  $m_{AB} = m_{CD}$

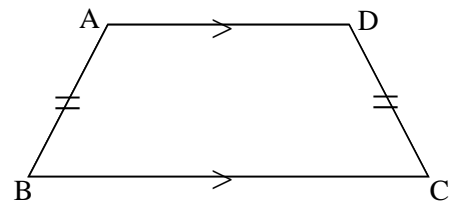
#### Perpendicular lines

The product of the gradients of two perpendicular lines is  $-1$ . If  $AB \perp CD$ , then  $m_{AB} \times m_{CD} = -1$

The properties of quadrilaterals are extremely important for Analytical Geometry and need to be mastered. These properties will now be summarised.

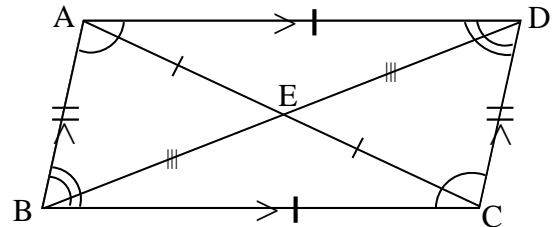
### Trapezium

- Two sides are parallel.



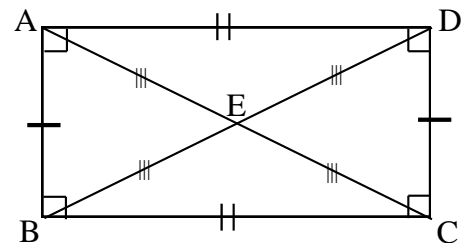
### Parallelogram

- Opposite sides parallel and equal.
- Opposite angles equal.
- Diagonals bisect each other.



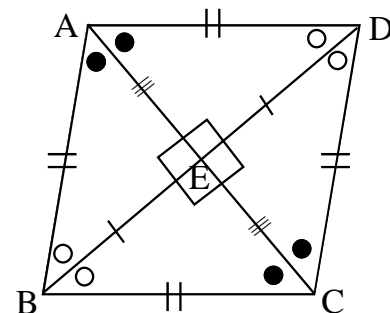
### Rectangle

- Opposite sides parallel and equal in length.
- Diagonals are equal in length and bisect each other.
- Interior angles are right angles.



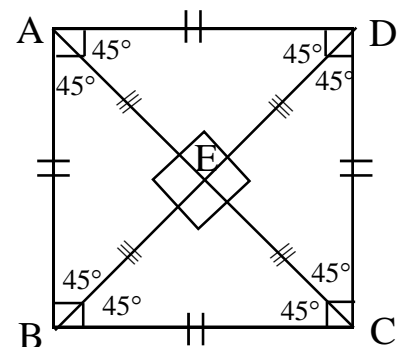
### Rhombus

- Opposite sides are parallel.
- All sides equal in length.
- Diagonals bisect each other at right angles.
- Diagonals bisect the opposite angles.



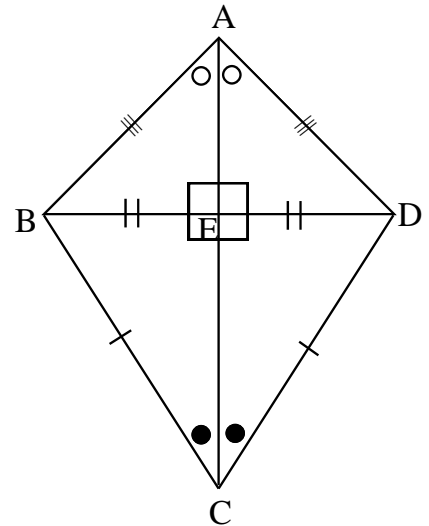
### Square

- Opposite sides parallel.
- All sides equal in length.
- Diagonals are equal in length.
- Diagonals bisect each other at right angles.
- Interior angles are right angles.
- Diagonals bisect interior angles (each bisected angle equals  $45^\circ$ )



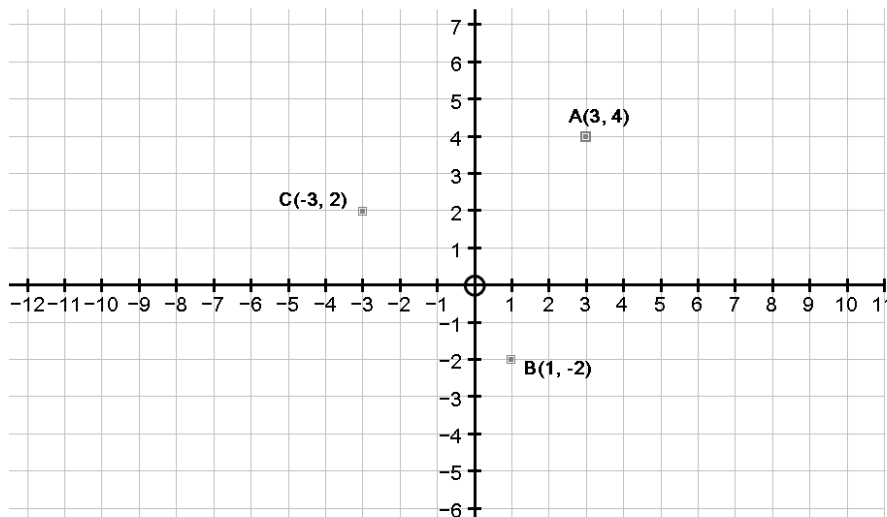
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- Adjacent pairs of sides are equal in length
- The longer diagonal bisects the opposite angles.
- The longer diagonal bisects the other diagonal.
- The diagonals intersect at right angles.



**X-ample Questions**

**Question 1**



- Determine the gradient of line  $BC$ . (2)
- Hence or otherwise determine the angle which  $BC$  makes with the positive  $x$ -axis (the angle of inclination). (2)
- Determine  $t$  if  $D(t;7)$  is collinear with  $B$  and  $C$ . (3)
- Determine  $p$  if  $E(5; p)$  if  $A$  is the mid-point of  $BE$ . (3)
- Determine the equation of the perpendicular bisector of  $BC$ . (4)
- Determine the coordinates of point  $D$  if  $ABCD$  (in that order) is to be a parallelogram. (2)

### Question 2

Calculate the gradients of AB and CD in each case and state whether AB and CD are:

(i) parallel      (ii) perpendicular      (iii) neither parallel or perpendicular

a)      A(-3;5), B (5; -1), C (-2;-1) and D (1;3)      (3)

b)      A(-2;-1), B (2;4), C(-3;-1) and D(0;3)      (3)

### Question 3

Calculate the value of x if A(-2;3), B(1;4), C(-4;1) and D(x;4) and B,C and D are collinear.      (4)

### Question 4

A(5;5), B(-7;1) and C(1;-7) are the vertices of triangle ABC.

a)      Show that ABC is an isosceles triangle.      (4)

b)      Determine the equation of the circle with the centre at the origin and passing through point A.      (3)

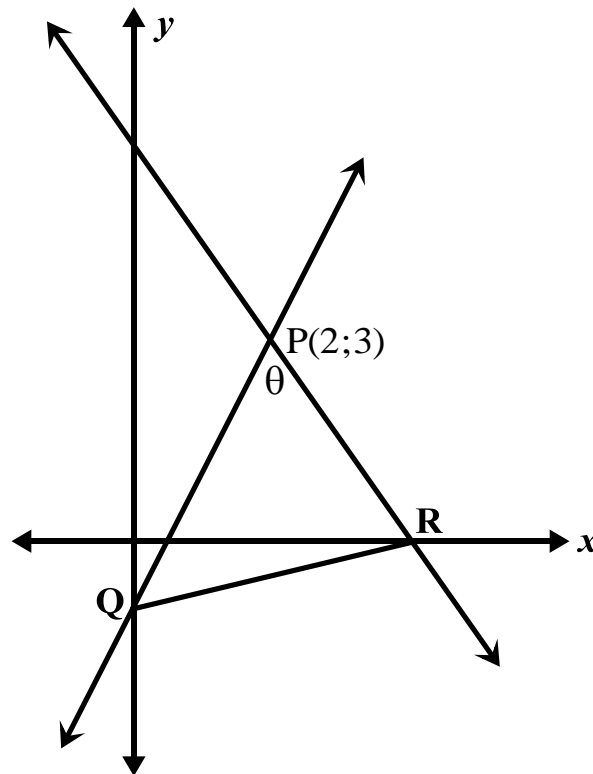
c)      Determine the equation of the perpendicular bisector of BC.      (4)

d)      Determine the equation of the line through A parallel to BC.      (3)

## X-ercises

### Question 1

The straight line through PQ with equation  $2x - y = 1$  cuts the  $y$ -axis at Q.  
The straight line through PR with equation  $3x + 2y = 12$  cuts the  $x$ -axis at R.  
The coordinates of P are (2;3).



### Determine:

- |     |   |     |
|-----|---|-----|
| (a) | The coordinates of Q and R.   | (2) |
| (b) | The equation of QR.   | (3) |
| (c) | The coordinates of point S, the midpoint of QR.   | (2) |
| (d) | The equation of PS.   | (1) |
| (e) | The size of angle $\theta$ rounded off to the nearest degree.<br>(Use the diagram for indicating angles). | (6) |
| (f) | The area of $\triangle PQR$ rounded off to two decimal places.  | (6) |