SESSION 9: TRIGONOMETRY: SPECIAL ANGLES & IDENTITIES

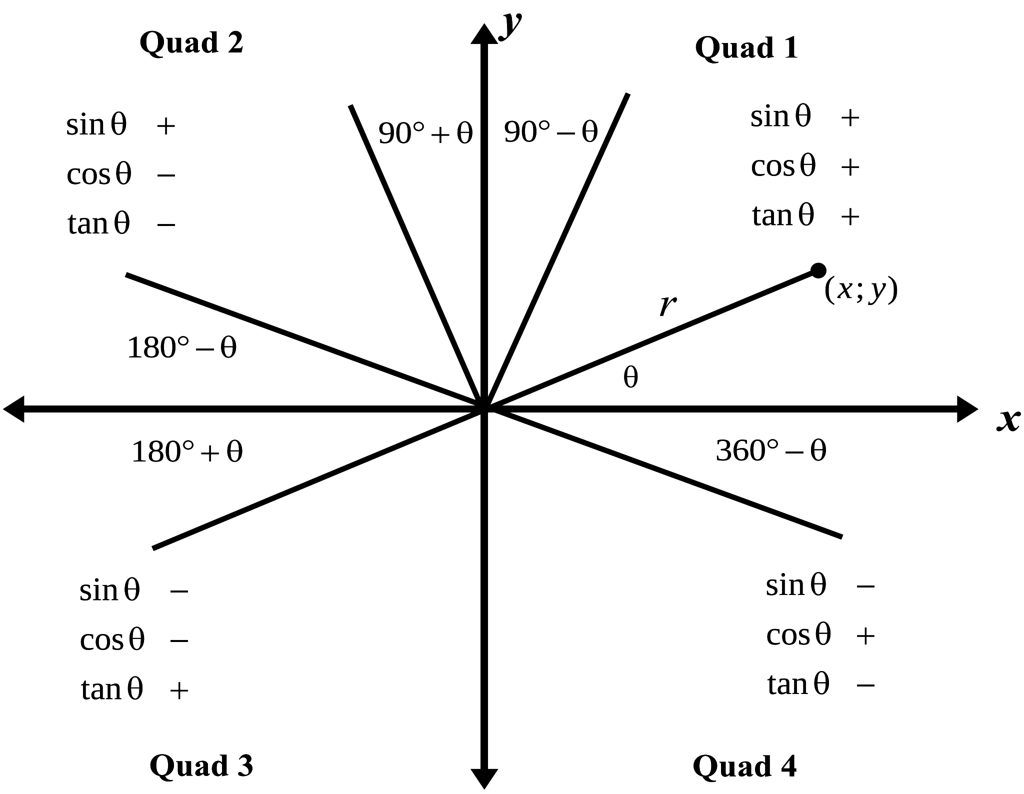
Key Concepts

In this session we will focus on summarising what you need to know about:

- Special angles and reduction formulae.
- Pythagoras questions and calculator work.
- Identities and general solution.

X-planation

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$



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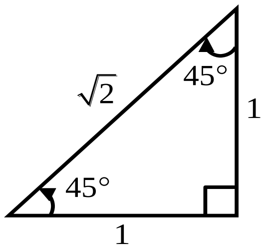
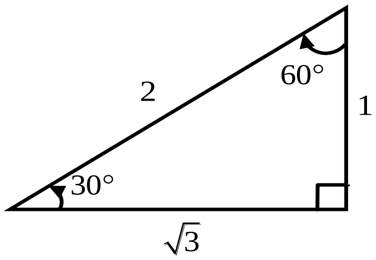
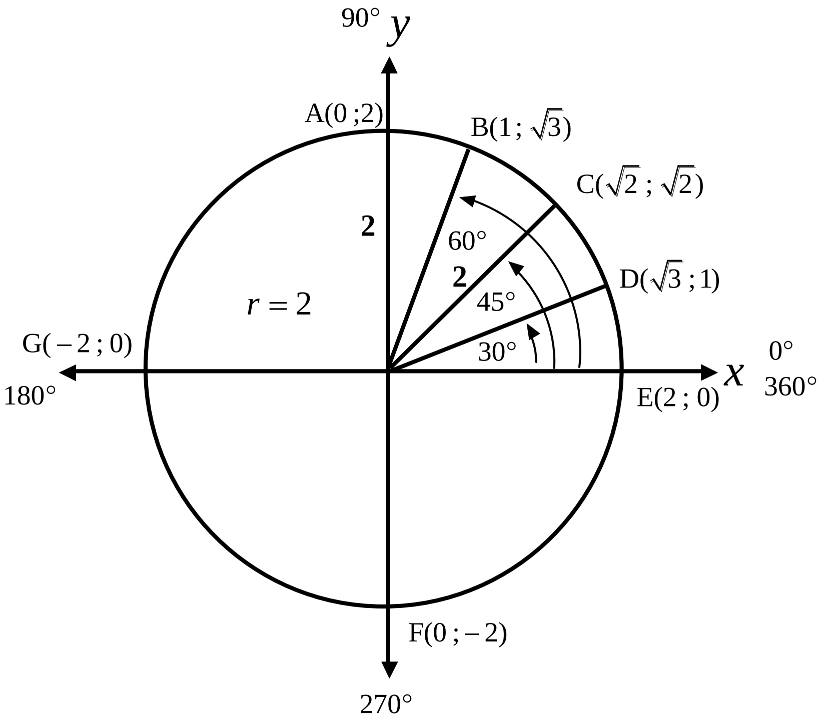
Special Angles







Identities

$$\cos^2\theta + \sin^2\theta = 1 \qquad\qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

Tips for proving identities:

1. Change all into sin and cos.
2. Do not cancel over a + and -.
3. Remember a common denominator.
4. Use square identities if you can.
5. Keep looking at the RHS.

Tips for general solution:

- Isolate the sin, cos or tan.
  Remember:
- $If\ cos(A) = b\ THEN\ (A) = \mp cos^{-1}b + k360°, k \in Z$
- $If\ \sin(A) = d\ THEN\ \ (A) = sin^{-1}d + k360°\ or (A) = 180° - sin^{-1}d + k360°, k \in Z$
- $If\ \tan(b) = a\ THEN\ (B) = tan^{-1}a + k180°\ k \in Z$

X-ample Questions

Question 1

If $x = 21{,}1°$ and $y = 47{,}5°$, calculate correct to 2 decimal places:

1.1 $\cos 2x$ (2)

1.2 $\sin(y - 2x)$ (2)

1.3 $\sqrt{\tan y} + \tan^2 y$ (3)

Question 2

2.1 Show that: $\dfrac{\cos(360° - x)\tan^2 x}{\sin(x - 180°)\cos(90° + x)} = \dfrac{1}{\cos x}$ (5)

2.2 Hence, calculate without the use of a calculator, the value of:

$\dfrac{\cos 330° \tan^2 30°}{\sin(-150°)\cos 120°}$ (Leave answer in surd form) (2)

2.3 Simplify without using a calculator:
$\cos^2(180° + x)[-\tan(360° - x).\cos(90° - x) - \sin(90° - x).\cos 180°]$ (9)

2.4 Show how you can demonstrate that the value of $\tan 300° = -\sqrt{3}$, without using a calculator. (2)

2.5 Simplify as far as possible:

2.5.1 $\dfrac{\tan(-\theta).\sin(90° - \theta)}{\sin(180° + \theta) + 3\sin(180° - \theta)}$ (6)

2.5.2 $3 + 2\cos^2\beta + 2\sin^2\beta$ (2)

Question 3

Given $\tan x = \sqrt{5}$ where $x \in [90°; 270°]$ and $\sqrt{2}\cos y + 1 = 0$ where $\sin y > 0$.

Determine the following using relevant diagrams:

3.1 $\dfrac{\sin^2 x}{\cos^2 y}$ (5)

3.2 $\cos(180° + x)$ (3)

Question 4

If $2\sin x - \sqrt{3} = 0$ and $\cos x < 0$, find the following using a relevant diagram:

4.1 $\cos^2 x$ (4)

4.2 $\dfrac{\sin(180° + x)}{\tan(360° - x)}$ (3)

4.3 $\cos(90° - x).\cos 30°$ (3)

Question 5

If $\tan A = \dfrac{p}{k}$ and $A < 90°$, find the following in terms of *p* and *k*.

5.1 $\sin A$ (2)

5.2 $\dfrac{1}{k}.\cos A$ (2)

Question 6:

If $\cos P = \dfrac{-3}{4}$ and $\sin P > 0$, use a sketch (no calculator) to find the value $\sqrt{7}\cos(90° + P)$. (4)

Question 7

Prove that:

7.1 $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$ (4)

7.2 $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = \dfrac{2}{\sin\theta}$ (5)

7.3 $\dfrac{\cos(180° - A)\tan(180° + A)\sin(360° - A)}{(\sin^2 A + \cos^2 A)\tan^2 A} = 1 - \sin^2 A$ (7)

7.4 $(\tan\theta + 1)^2 + (\tan\theta - 1)^2 = \dfrac{2}{\cos^2\theta}$ (5)

Question 8

8.1 Evaluate $\beta \in [-180°;360°]$ correct to one decimal place if $3\sin 2\beta = 2$. (7)

8.2 Find the general solution correct to 1 decimal place if: $\cos(2x + 20°) = 0.85$ (5)

8.3 Determine the general solution of the equation $\cos^2\theta = 2 - \cos\theta$ (5)

X-ercises

Question 1

1.1 Evaluate the following without using a calculator:

$$\frac{\sin 120°.\tan(-30°)}{\cos^2 240°}$$ (5)

1.2 Simplify into a single term:

$$\frac{\sin(180° + \theta)}{\tan(\theta - 180°).\cos(180° - \theta)} - \sin^2(90° - \theta)$$ (6)

Question 2

If $\tan 27° = k$, express the following in terms of *k* (a sketch may help):

2.1 $\sin 27°$ (5)

2.2 $\sin 63°$ (3)

2.3 $\tan(-27°)$ (2)

2.4 $\cos^2 387°$ (2)

Question 3

If $\alpha = 78{,}3°$ and $\beta = 18{,}5°$, use a calculator to find the value of (2 decimal places):

3.1 $\cos(\alpha - 5\beta)$ (2)

3.2 $3\tan\alpha + 4\sin\frac{\beta}{2}$ (2)

Question 4

4.1 Prove that

$$\frac{\cos x}{\sin x} - \tan x = \frac{2\cos^2 x - 1}{\sin x.\cos x}$$ (5)

4.2 Determine the general solution if $\tan(2x - 10°) = 0.58$ (5)