

SESSION 11: FINANCIAL MATHS

Key Concepts

In this session we will focus on summarising what you need to know about:

- Different compounding periods
- Nominal and annual effective rates
- Depreciation
- Linear Depreciation
- Reducing Balance Depreciation

X-planation

Different compounding periods

Interest can be quoted per annum but calculated over different time periods during a year. Interest can be calculated:

annually: Once per year (usually at the end of the year)

semi-annually: Twice per year (every six months/half-yearly)
Divide r by 2 and multiply n by 2

quarterly: Four times per year (every three months)
Divide r by 4 and multiply n by 4

monthly: Twelve times per year (every one month)
Divide r by 12 and multiply n by 12

The formula to find A when given P is as follows:

$$A = P(1+i)^n$$

The formula to find P when given A is as follows:

$$P = A(1+i)^{-n}$$

Nominal and annual effective rates

A nominal rate is an annual rate which does not take into consideration the effect of different compounding periods. For example, 15% per annum compounded monthly is a nominal rate.

It is possible to determine an annual effective rate compounded annually that will yield the same accumulated amount as the annual rate compounded monthly.

The formula to calculate the effective annual rate given a nominal rate is:

$$1 + i_{\text{eff}} = \left(1 + \frac{i_{\text{nom}}}{n}\right)^n$$

where:

i_{nom} = nominal rate

i_{eff} = effective rate (annual)

n = number of compoundings per year

Depreciation

When equipment loses its value over a given time period, we say that the equipment is depreciating in value over time. There are two types of depreciation:

Linear Depreciation and Reducing Balance Depreciation.

Linear Depreciation

With linear depreciation, equipment is depreciated by a percentage of its original value. It works in the same way as simple interest, but the value decreases rather than increases as with simple interest.

The formula for linear depreciation is

$$A = P(1 - i.n)$$

Reducing Balance Depreciation

With reducing-balance depreciation, equipment is depreciated by a percentage of its previous value. It works in the same way as compound interest, but the value decreases rather than increases as with compound interest.

The formula for Reducing Balance Depreciation is

$$A = P(1 - i)^n$$

X-ample Questions

Question 1

- (a) Peter invests R4000 for 6 years at an interest rate of 16% per annum compounded monthly. Calculate the value of the investment at the end of the six year period. (4)
- (b) What amount must be invested for 2 years at an interest rate of 10% per annum compounded half-yearly in order to receive R10 000? (4)

Question 2

Tshepo invests R40 000 for five years at 16% per annum compounded monthly.

- (a) Calculate the future value of the investment using the nominal rate. (4)
- (b) Convert the nominal rate of 16% per annum compounded monthly to the equivalent effective rate (annual). (4)
- (c) Now use the annual effective rate to show that the same accumulated amount will be obtained as when using the nominal rate. (4)

Question 3

Siyabonga wants to sell his car in five years time. The rate of depreciation is 14% per annum and the current value of the car is R60 000. Calculate the book value of the car in five years' time if depreciation is based on:

- (a) the straight-line method? (3)
- (b) the reducing-balance method? (3)

Question 4

A laptop cost R9 000 and, after four years, has a scrap value of R2000. Find the annual depreciation rate if it is calculated using:

- (a) the straight line method. (3)
- (b) the reducing balance method. (3)

Question 5

Sibongile invests R16000 for four years at 10% per annum simple interest. Thereafter, she invests the accumulated amount for another two years at 12% per annum compounded monthly. Calculate how much money she will have saved at the end of the six year period. (4)

Question 6

Refilwe deposits R30000 into a savings account. The interest rate for the first three years is 7% per annum compounded half-yearly. Thereafter, the interest rate changes to 8% per annum compounded quarterly. Calculate the value of the investment at the end of the tenth year. (7)

Question 7

Malibongwe wants to have saved R4 000 000 in eight years' time. How much

must he invest now if the interest rate for the first six years is 6% per annum compounded monthly and 8% per annum compounded quarterly for the remaining two years? (7)

Question 8

Simon deposited R1000 into a bank. Three years later, he deposited a further R2000 into the bank. The interest rate for the five year savings period was 18% per annum compounded annually. Calculate the future value of his investment at the end of the savings period. (5)

X-ercises

Question 1

- (a) Determine which of the following two savings options is a better investment over a period of one year if the interest is calculated at:
- A. 15% per annum compounded monthly
 - B. 17% per annum compounded quarterly (5)
- (b) Joseph invested an amount of money six years ago. Now, after six years, it is worth R1 200 000. The interest rate for the savings period was 18% per compounded monthly. What was the amount that was originally invested six years ago? (3)

Question 2

Tamara invests R24 000 at 14% per annum compounded quarterly for a period of twelve years.

- (a) Calculate the future value of the investment using the nominal rate. (4)
- (b) Convert the nominal rate of 14% per annum compounded quarterly to the equivalent effective rate (annual). (4)
- (c) Use the annual effective rate to show that the same accumulated amount will be obtained as when using the nominal rate. (4)

Question 3

- (a) The office computers for a small business are currently worth R40 000. Calculate the value of these computers after 6 years if the rate of depreciation is 16% per annum calculated on a linear basis. (3)
- (b) A motor vehicle currently has a book value of R6000. The rate of depreciation was 14% per annum using the reducing balance method. Calculate the original price of the motor vehicle if it was bought 5 years

ago. (3)

- (c) A photocopy machine costs R40 000 and has a scrap value of R8000 after 8 years. Find the annual rate of depreciation if it is calculated using:
- (i) the straight-line method. (3)
 - (ii) the reducing-balance method. (3)

Question 4

Michael deposits a gift of R20 000 into a savings account in order to save for an overseas trip in six years' time. The interest rate for the savings period is 8% per annum compounded monthly.

- (a) How much money will he have saved at the end of the six year period? (4)
- (b) Suppose that at the end of the second year, he withdraws R4000 from the account. How much money will he then have saved at the end of the six year period? (7)
- (c) Suppose that at the end of the third year, he adds R3000 to the savings. How much money will he have then saved at the end of the six year period? (7)

Question 5

Simphiwe deposits Rx into a unit trust savings account. Two years later she deposits a further $R2x$ into the account. Three years after this she deposits $R3x$ into the account. The interest rate for this five year period is 18% per annum compounded monthly. She receives R60 000 at the end of the five year period. Calculate the value of x . (7)

Question 6

Brian takes out a loan to finance the purchasing of a new DVD player. He repays the loan by means of a payment of R5000 four years after the granting of the loan. Two years later he repays a final amount of R6000. The interest rate during the first four years of the loan is 16% per annum compounded quarterly. For the remaining two years the interest rate changes to 14% per annum compounded half-yearly. How much money did Brian originally borrow? (7)