# ANALYTICAL GEOMETRY II 03 JUNE 2013

## Lesson Description

In this lesson we:

- Work through a number of examples to master the major key concepts in Analytical Geometry.

## Key Concepts

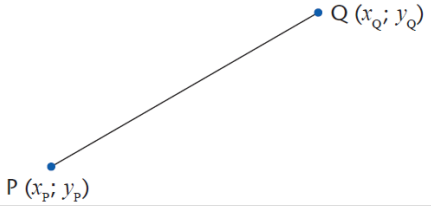
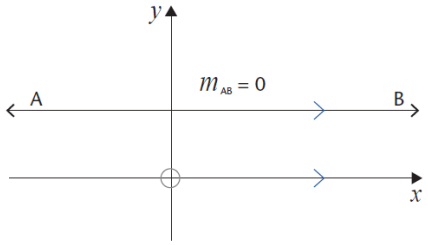
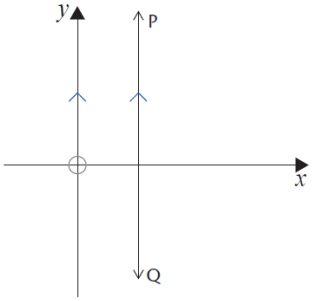
1. Gradient, Parallel Lines, Perpendicular Lines
2. Equation of a Line
3. Distance of a Line Segment and Midpoint of a Line Segment.
4. Inclination
5. Properties of Geometrical Figures

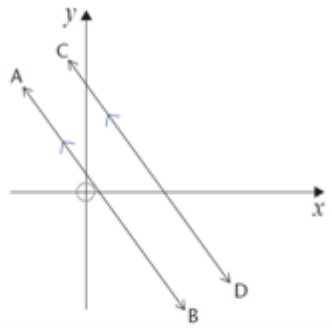
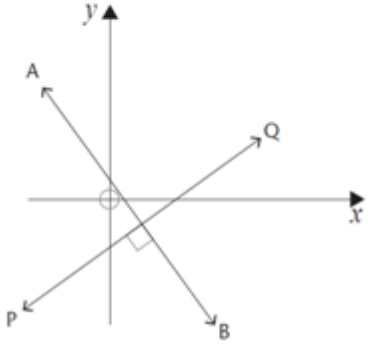
## Gradient

*(From Grade 11, Clever Keeping Maths Simple, Macmillan, Page 89)*

### Summary of the gradient of a straight line

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} \; ; \; x_B \neq x_A$$

| | |
|---|---|
| 1.   | If P and Q are any two points on a non-vertical straight line, then: $m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P} = \frac{y_P - y_Q}{x_P - x_Q}$ |
| 2.   | The gradient of a **horizontal** straight line is **zero**. $m_{AB} = 0$ |
| 3.   | The gradient of a **vertical** straight line is **undefined**. $m_{PQ}$ is undefined |

| | |
|---|---|
| 4.   | If two straight lines are **parallel**, then their **gradients** are **equal**.<br>If AB ∥ CD, then:<br>$m_{AB} = m_{CD}$ |
| 5.   | If two non-vertical straight lines are **perpendicular**, then the **product** of their **gradients** is $-1$.<br>If AB ⊥ PQ, then:<br>$m_{AB} \cdot m_{PQ} = -1$ or $m_{AB} = \frac{-1}{m_{PQ}}$ |

## Equation of a Straight Line

*(From Grade 11, Clever Keeping Maths Simple, Macmillan, Page 96)*

| General forms of a straight line equation | Use this when you know: |
|---|---|
| $y = mx + c$<br>gradient $y$-intercept | The **gradient** and the **$y$-intercept** |
| $y - y_1 = m(x - x_1)$<br>gradient<br>coordinates of a point on the line | The **gradient** and the **coordinates** of at least **one point** on the graph. |

## Distance of a Line Segment and Midpoint of a Line Segment

*(From Grade 11, Clever Keeping Maths Simple, Macmillan, Page 80)*

### The distance between two points

The distance between the points $P(x_P; y_P)$ and $Q(x_Q; y_Q)$ is given by the formulae:

$$PQ = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2} \text{ or } PQ^2 = (x_Q - x_P)^2 + (y_Q - y_P)^2$$
$$\text{or } PQ = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}$$

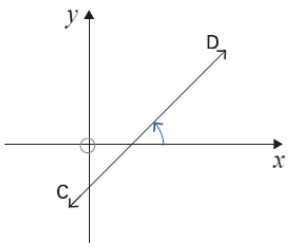
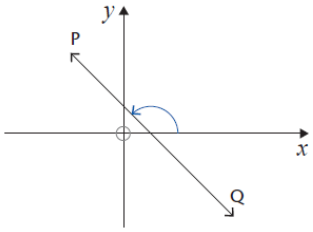
## The midpoint of a line segment

The coordinates of the midpoint M($x_M$; $y_M$) of a line segment PQ are given by:

$$x_M = \frac{x_P + x_Q}{2}; \; y_M = \frac{y_P + y_Q}{2}$$

## Inclination of a Line

*(From Grade 11, Clever Keeping Maths Simple, Macmillan, Page 91)*

| I | II |
|---|---|
|   |   |
| • $\tan\theta = m_{CD}$<br>• Note that the gradient of CD is **positive** and thus $\tan\theta$ is **positive**.<br>• Here $\theta$ is an **acute** angle. | • $\tan\theta = m_{PQ}$<br>• Note that the **gradient** of PQ is **negative** and thus $\tan\theta$ is negative.<br>• Here $\theta$ is an **obtuse** angle. |

By definition, the inclination $\theta$ is such that $0° \leq \theta < 180°$. Thus we can conclude that if:

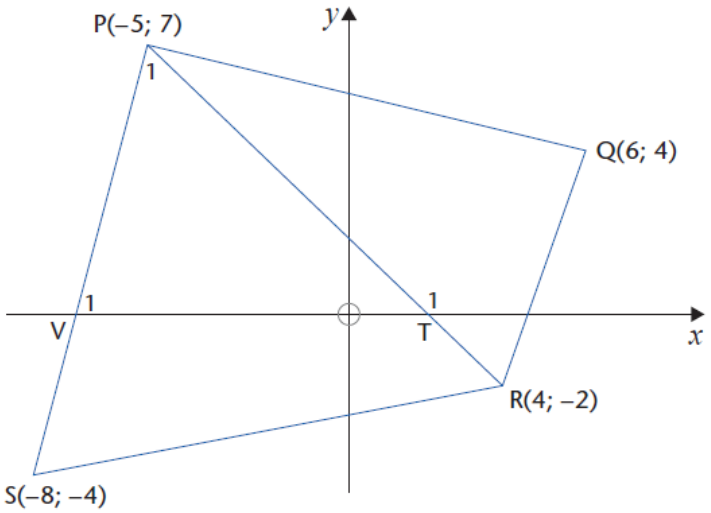
- $\tan\theta$ is positive, then $0° \leq \theta < 90°$.
- $\tan\theta$ is negative, then $90° < \theta < 180°$.

Note that we exclude 90° since $\tan\theta$ is undefined for $\theta = 90°$.

## Example

*(From Grade 11, Clever Keeping Maths Simple, Macmillan, Page 104, Exercise 4.6, Question 2)*

P(−5; 7), Q(6; 4), R(4; −2) and S(−8; −4) are the vertices of quadrilateral PQRS.



a.) Prove that PS ⊥ PQ.
b.) Determine the equation of PR.
c.) Calculate the coordinates of S, the *y*-intercept of PR.
d.) Calculate the size of $\hat{T}_1$.
e.) Determine the size of $\hat{P}_1$ correct to one decimal place.

### A summary of the concepts in this chapter

*(From Grade 11, Clever Keeping Maths Simple, Macmillan, Page 91)*

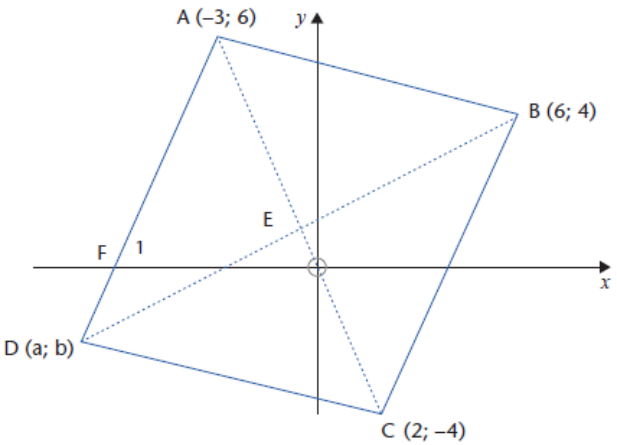
| | |
|---|---|
| 1. **The distance formula**<br>Used to find the length of a line segment | $AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$ or<br>$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$ |
| 2. **The midpoint formula**<br>M is the midpoint of line segment PQ | $x_M = \frac{x_P + x_Q}{2}$; $y_M = \frac{y_P + y_Q}{2}$ |
| 3. **The gradient of a straight line** | |
| 3.1 If given the coordinates of two points on the straight line: | $m = \frac{y_2 - y_1}{x_2 - x_1}$ |
| 3.2 If given two parallel straight lines: | $m_1 = m_2$ |
| 3.3 If given two perpendicular straight lines: | $m_1 \times m_2 = -1$ or $m_1 = -\frac{1}{m_2}$ |
| 3.4 If given the inclination angle: | $m = \tan\theta$ |
| 4. **The equation of a straight line**<br>You always need to know:<br>• the gradient<br>• the coordinates of a point on the line | $y - y_1 = m(x - x_1)$ |

## Questions

### Question 1

*(From Grade 11, Clever Keeping Maths Simple, Macmillan, Page 105, Exercise 4.6, Question 4)*

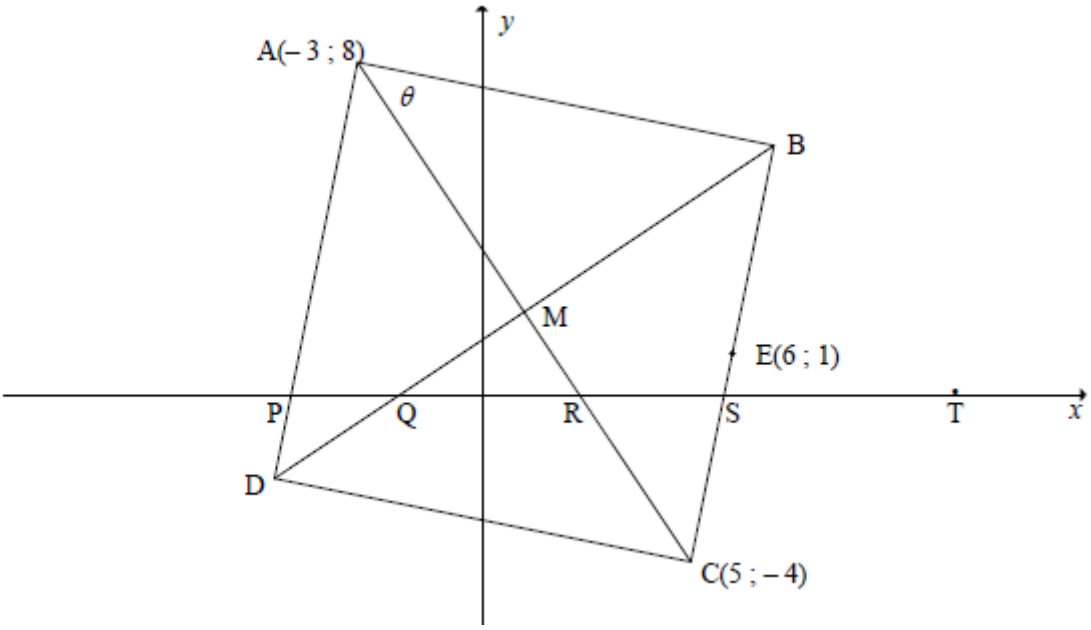
A(−3; 6), B (6; 4), C(2; −4) and D(*a*; *b*) are the vertices of parallelogram ABCD.




a.) Write down the coordinates of E, the midpoint of AC.
b.) Hence determine the coordinates of D, the fourth vertex of the parallelogram.
c.) Determine the equation of the diagonal AC.
d.) Calculate the inclination of AC correct to one decimal place.
e.) Hence calculate the size of $D\hat{A}C$, correct to one decimal place.

## Question 2

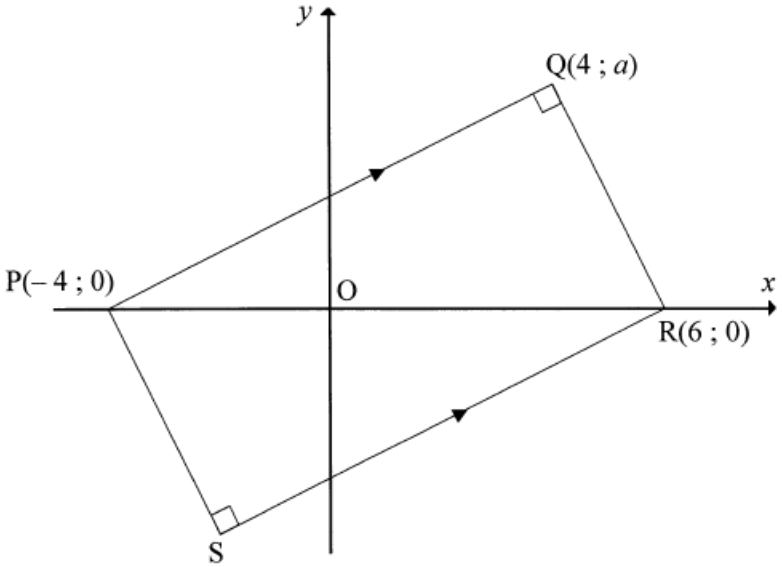
ABCD is a rhombus with A(-3 ; 8) and C(5 ; -4). The diagonals of ABCD bisect each other at M. The point E(6 ; 1) lies on BC.




a.) Calculate the coordinates of M.
b.) Calculate the gradient of BC.
c.) Determine the equation of the line AD in the form $y = mx + c$
d.) Determine the size of $\theta$, that is $B\hat{A}C$. Show ALL calculations.

## Question 3

In the diagram below, PQRS is a rectangle with vertices P(-4 ; 0), Q(4 ; a), R(6 ; 0) and S. Q lies in the first quadrant.




a.) Show that a = 4

b.) Determine the equation of the straight line passing through the points S and R in the form $y = mx + c$

c.) Calculate the coordinates of S.

d.) Calculate the length of PR.

e.) Rectangle PQRS undergoes the transformation (x ; y) → (x + k; y + l) where k and l are numbers. What is the minimum value of k + l so that the image of PQRS lies in the first quadrant (that is, x ≥ 0 and y ≥ 0)?