EUCLIDEAN GEOMETRY II

29 JULY 2013

Lesson Description

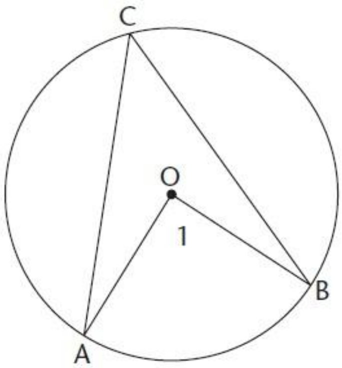
In this lesson we:

- Work with 4 theorems in riders found in Circle Geometry:
  - The Angle at the Centre
  - The Diameter
  - Angles in the Same Segment
  - Cyclic Quadrilaterals: Opposite Angles and Exterior Angle.

Key Concepts

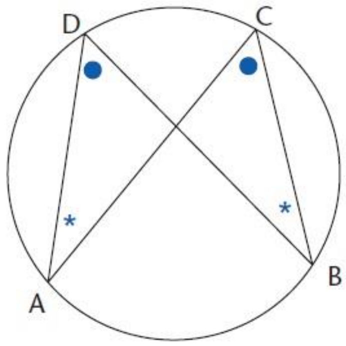
Summary of Theorems

* $\angle$ at centre $= 2 \times \angle$ at circumference



$\hat{O}_1 = 2\hat{C}$

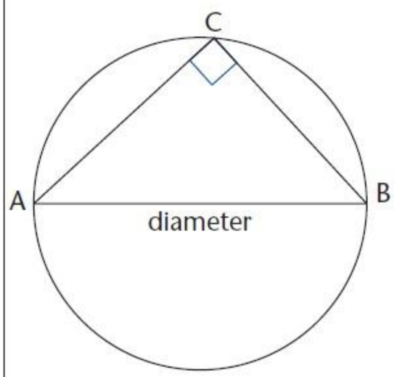
* $\angle$s in same segment are equal



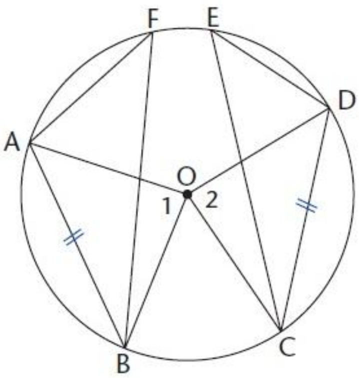
$\hat{D} = \hat{C}$

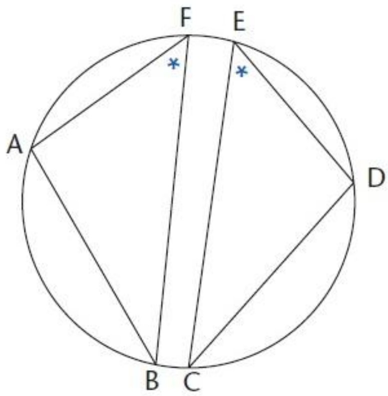
$\hat{A} = \hat{B}$

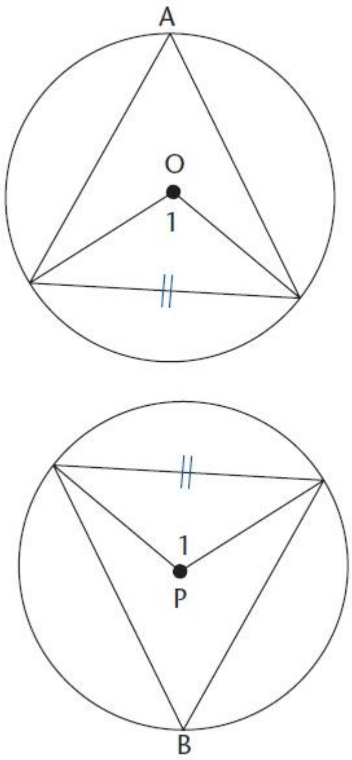
$\angle$ in semicircle $= 90°$



$\hat{C} = 90°$

<table>
<tr>
<td>Equal chords subtend equal ∠s at:
• circumference
• centre


$\hat{O}_1 = \hat{O}_2$
$\hat{F} = \hat{E}$</td>
<td>Equal ∠s at circum (or centre) are subtended by equal chords


AB = CD</td>
<td>Equal chords in equal circles subtend equal ∠s at:
• circumference
• centre


$\hat{O}_1 = \hat{P}_1$
$\hat{A} = \hat{B}$</td>
</tr>
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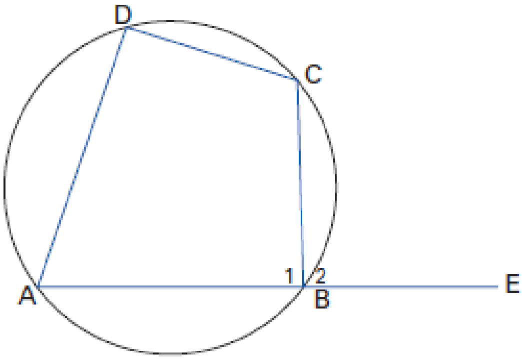
Cyclic Quadrilaterals

- A quadrilateral is said to be cyclic if all four of its vertices lie on the circumference of a circle.
- Four or more points are said to be concyclic if a circle can be drawn to pass through all the points.
- ABCD is a cyclic quadrilateral since the vertices A, B, C and D lie on the circumference of the circle.
- If AB is produced to E, then $\hat{B}_2$ is said to be an exterior angle of the cyclic quadrilateral.
- Remember that supplementary angles are angles that add up to 180°.

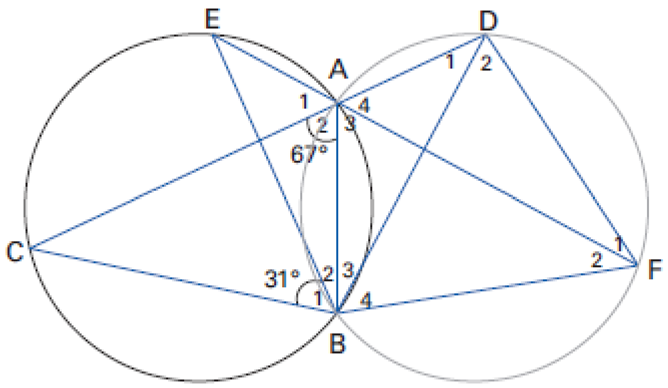


- The relationship between the opposite angles of a cyclic quadrilateral
- The relationship between the exterior angle of the cyclic quadrilateral and the opposite interior angle.
- How to prove that a quadrilateral is cyclic

Questions

Question 1

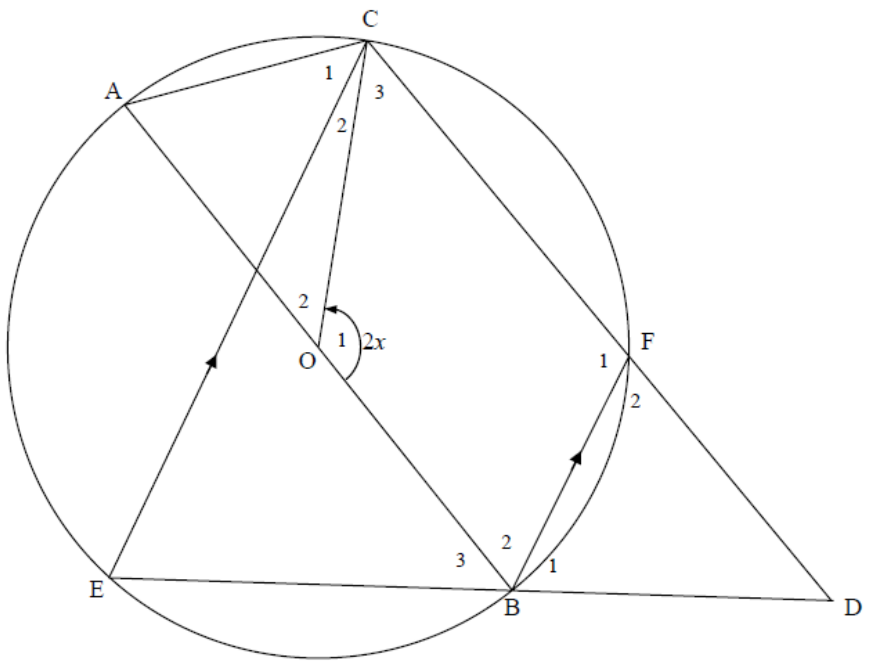
Two circles intersect at A and B. EAF and DAC are double chords.
$\hat{A}_2 = 67°$ and $\hat{B}_1 = 31°$.
Calculate the size of $D\hat{F}B$.



Question 2

(Adapted from DBE, March 2012)

In the diagram below, O is the centre of the circle. AB is a diameter of the circle. Chord CF produced meets chord EB produced EB produced at D. Chord EC is parallel to chord BF. CO and AC are joined. Let $\hat{O}_1 = 2x$



a.) Determine, in terms of x, the size of $\hat{F}_1$. (4)

b.) Prove that DF = BD. (4)

c.) Show that $\hat{C}_1 = \hat{C}_3$. (4)

Question 3

Two circles intersect at B and A. CBD is a double chord. CSE and DTE are drawn to meet BA produced in E. SA and AT are drawn.

Let $\hat{B}_1 = x$ and $\hat{A}_4 = y$.

Prove that:

a.) SATE is a cyclic quadrilateral.
b.) STDC is a cycle quadrilateral.