

Sequences and Series

Key Concepts

In this session we will focus on summarising what you need to know about:

- How to find the nth term of a quadratic sequence. (constant second difference)
- Finding a term of an arithmetic sequence.
- Finding a term of a geometric sequence.
- Using simultaneous equations to find a sequence.
- Develop a formula for the sum of a arithmetic and geometric series.

Terminology & definitions

Sequence: Listing of terms. i.e. $T_1; T_2; T_3; T_4 \dots \dots \dots$

Series: Adding terms of a sequence i.e. $T_1 + T_2 + T_3 + T_4 \dots \dots \dots$

Arithmetic Sequence: Has a constant first difference $d = T_2 - T_1$

Quadratic Sequence: Has a constant second difference.

Geometric Sequence: Has a constant ratio. $r = \frac{T_2}{T_1}$

Constant- remains the same

Symbols, Units & Equations

Common difference – d

First term – a

Common ratio - r

Formula Sheet

$$\sum_{i=1}^n [a + (i-1)d] = \frac{n}{2} [2a + (n-1)d]$$

$$\sum_{i=1}^n ar^{i-1} = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1 - r} ; -1 < r < 1$$

$$T_n = an^2 + bn + c$$

X-planation

- Even though formulas are given, the best place for them to be is in your head.
- The formula sheet should only be used if you forget a formula.
- Code when needed.
- Always use the correct formula.

X-ample Questions

1. The first four terms of a sequence are: 1; 10; 25; 46;...;
 - (a) Write down the next two terms. (2)
 - (b) Determine the formula T_n for the sequence. (5)
 - First term = $a+b+c$
 - First first difference = $3a+b$
 - Second constant difference = $2a$
 - Formula for quadratic formula is $T_n = an^2 + bn + c$
2. Given the sequence: 3 ; 6 ; 3 ; 17 ; 3 ; 34 ; 3 ; 57
Determine the 40th term of the sequence. (7)
3. Write down the next term of the following arithmetic sequence:
 $2x; 2x+2; \dots\dots\dots$ (1)
4. Write down the next term of the following geometric sequence:
 $x ; \sqrt{x} ; \dots\dots\dots$ (1)
5. Determine the 7th term of the following sequence:
 $\frac{1}{64} ; \frac{3}{32} ; \frac{9}{16} ; \frac{27}{8} ; \dots\dots\dots$ (3)
6. The following sequence of numbers is given: 3; 7; 11; ..
 - a) Is 33 a number in this sequence? (1)
 - Remember a term is always a natural number. We can not have the three and a half term.
 - b) Write down the first number in the sequence greater than 33. (1)
7. The second term of a geometric series is $\frac{1}{10}$, the fifth term is $\frac{27}{1250}$.
Find the value of the first term and the constant ratio. (5)
 - Always code the question i.e. $T_2 = \frac{1}{10}$
 - Use the correct formulas
8. $t+1; 1-t; 2-5t$ are the first three terms of a geometric sequence
 - a) Calculate the value/s of t . (5)
 - b) Calculate the value of the common ratio. (2)
9. Consider the sequence 1;2;3;4;5;6;7;8 ;9; ... ;41
 - a) Write down the next THREE terms after 9. (3)
 - b) Calculate the number of terms in this sequence. (5)
10. The second and fourth term of a converging geometric series are respectively 36 and 16. All the terms are positive.
 - a) Determine the constant ratio (3)
 - b) The first term (1)

X-exercise

1. $-\frac{3}{2}$; 3; -6 form a geometric sequence, determine an expression for the n^{th} term. (2)
2. $-\frac{3}{4}$; a; b; $\frac{2}{9}$ form a geometric sequence, find a and b. (4)
3. For the geometric sequence with $T_5 = 486$ and $r = 3$:
 - a) Find the rule for the n^{th} term (4)
 - b) If the last term in the sequence is 118 098, use the rule in (a) to calculate how many terms there are in the sequence? (4)
4. Give the rule for the following sequence: 0 ; 10 ; 24 ; 42 ; 64... (4)
5. Given the sequence: -8; -2; 4;
 - a) State whether it is arithmetic, geometric or neither (1)
 - b) Write down the next three terms of the sequence (2)
 - c) Find an expression for the n^{th} term. (2)
6. Determine the value of the 15th term of the sequence: 1; 3; 6; 10; 15; 21; ... (3)
7. Given the arithmetic sequence 16; 11; 6; ...
 - a) Write down the next two terms. (2)
 - b) Find an expression for the n -th term. (2)
8. The first three terms of an arithmetic sequence are $(m - 2)$; $(2m - 6)$; $(4m - 8)$ Find the value of m . (2)
9. Consider the geometric sequence $-\frac{3}{2}$; 3 ; -6 ; ...
Which term in the sequence is equal to -96 (4)

Answers

1. $T_n = -\frac{3}{2}(-2)^{n-1}$
2. $a = \frac{1}{2}$ and $b = -\frac{1}{3}$
3. a) $T_n = 6 \cdot 3^{n-1}$
b) $n = 10$
4. $T_n = 2n^2 + 4n - 6$
5. a) Arithmetic
b) 10 ; 16 ; 22
c) $T_n = 6n - 14$
6. $T_n = \frac{1}{2}n^2 + \frac{1}{2}n$ so $T_{15} = 120$
7. a) 1 ; -4
b) $T_n = 21 - 5n$
8. $m = 2$
9. The seventh term is -96