Sequences and Series Part 2

Key Concepts

In this session we will focus on summarising what you need to know about:

- Working with arithmetic and geometric series
- How many terms are added to get a particular sum
- Sigma notation
- How to find a term if you are given a formula for the sum
- Convergence
- The sum to infinity
- Problems using sequences and series
- Diagramatic problems

Terminology & definitions

Convergence- tending towards a particular value.
Divergence- will continue to positive or negative infinity
Numerical-number value
Specific – a particular
Infinite – never ending
Midpoints – half way on a line

Key Concepts

Sigma notation

$$\begin{array}{c} \textit{to} \\ \downarrow \\ \textit{Sum} \rightarrow \sum_{n=1}^{10} T_n \leftarrow \textit{formula} \\ \uparrow \\ \textit{from} \end{array}$$

$$\sum_{n=1}^{10} T_n = T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 + T_8 + T_9 + T_{10}$$

Symbols, Units & Equations

$$\sum_{i=1}^{n}(a+(i-1)d) = \frac{n}{2}[2a+(n-1)d]$$

$$\sum_{i=1}^{n} ar^{i-1} = \frac{a(1-r^{n})}{1-r}$$

$$\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}$$

X-planation

- Always first establish what kind of series you are working with i.e.arithmetic or geometric. You must always use the correct formula or you will receive no marks.
- If you are given sigma notation always work out the first three terms so that you can decide what type of series.
- To decide how many terms must be summed

  $$\sum_{n=start}^{end} T_n = S_{end-start+1}$$

  So

  $$\sum_{n=5}^{13} T_n$$
  $$= S_{13-5+1}$$
  $$= S_9$$

- If given a formula for S_n and asked to find T_n then $T_n = S_n - S_{n-1}$
- The number of terms will always be a natural number so always discard negative of fractional answers.

X-ample Questions

1. How many terms of the series 2+6+10+ .... will give a sum of 800 (6)

- Establish what kind of series you are dealing with.
- Be careful you select the correct formula.

2. Calculate the value of x if: $\sum_{n=1}^{x}(10+2n) = 242$ (6)

- Always work out the first three terms so that you can decide what type of series.

3. Given that $\sum_{k=1}^{4}(2^k + m) = m - 3$ find the numerical value of m.

(4)

- Since there are only four substitutions work them all out.

4. Solve for n, the number of terms, if: $\sum_{k=1}^{n} 4(\frac{1}{2})^{k-1} = 7\frac{63}{64}$

(7)

5. If $S_n = n(n+1)$ represents the sum of the first n terms of a specific series. Determine the 10^{th} term.

(3)

6. For the geometric sequence with T= 486 and r = 3.

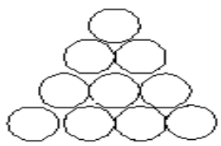
a) Find the rule for the n^{th} term. (4)

b) If the last term in the sequence is 118 098, use the rule in (a) to calculate how many terms there are in the sequence? (4)

c) Calculate the sum of the first 10 terms in the sequence. (3)

7. Fifty-five water pipes are to be stacked in a triangular pattern as shown in the diagram. Use an applicable formula to determine how many pipes must be placed on the bottom in order to have only one pipe in the top layer.

(4)



8. Consider the infinite geometric series:

$(x - 2)^2 + (x - 2)^3 + (x - 2)^4 + \ldots$

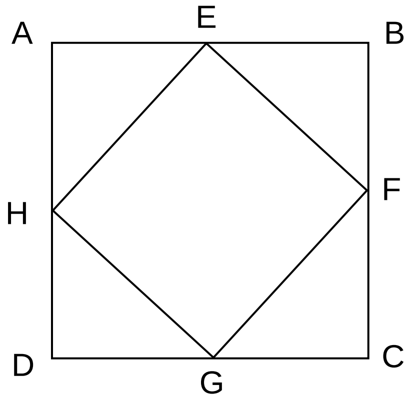
a) Write down the common ratio in terms of *x* (1)

b) Determine the value(s) of *x* for which the series converges (3)

c) If we assume that the series converges and that the sum of the series to infinity is $\frac{x-2}{3}$, calculate the value(s) of *x*. (5)

9. Calculate the value of $\sum_{k=1}^{\infty} 4(\frac{2}{3})^k$ (4)

10. ABCD is a square of side 10 cm.
Square EFGH is formed by joining the midpoints of the sides, as shown. The midpoints of those sides are joined to form a 3rd square, and so on.



a) Show that the sides of square EFGH are each $\sqrt{50}$ cm long. (2)

b) Show that the areas of successive squares form a geometric sequence and write down the 1st 3 terms. (4)

c) Calculate the sum of the areas of all the squares formed in this way. (3)

X-ercise

1. For which values of *x* does $\sum_{i=1}^{\infty}(2x-1)^i$ exist? (3)

2. Determine $\sum_{p=1}^{\infty}(x+2)^p$ when $x = -\frac{5}{2}$ (4)

3. If the sum of *n* terms is given by $S_n = \frac{n}{2}(1+n)$, find T_5. (3)

4. The first term of a geometric series is *p* and the second term is $p^2 + p$ ($p \neq 0$).
   i) Write down the common ratio in terms of p. (1)
   ii) Determine the values of *p* for which the series is convergent. (2)
   iii) If the series is convergent, calculate the sum to infinity. (3)

5. $S_n = 4n^2 + 1$ represents the sum of the first n terms of a series. Find the value of the second term of this series (3)

6. The first three terms of an arithmetic sequence are $(m-2)$; $(2m-6)$ and $(4m-8)\}$
   i) Find the value of m. (2)
   ii) Determine the sum of the first 20 terms. (3)

7. Calculate:
$$\sum_{i=1}^{\infty} 5.4^{-i}$$
(4)

8. Determine the smallest value of *K* for which $\sum_{n=2}^{k}(2n-3)$ exceeds 400. (6)

ANSWERS

1. $0 < x < 1$
2. $S_{\infty} = -\dfrac{1}{3}$
3. $T_5 = 5$
4. (i) p+1
   (ii) $-2 < p < 0$
   (iii) $S_{\infty} = -1$
5. $T_2 = 12$
6. (i) m=-2
   (ii) $S_{20} = -1220$
7. $S_{\infty} = \dfrac{5}{3}$
8. 21 terms