Saturday X-tra **X-Sheet: 8**

Inverses and Functions

Key Concepts

In this session we will focus on summarising what you need to know about:

- How to find an inverse.
- How to sketch the inverse of a graph.
- How to restrict the domain of a function so the inverse is a function.

Terminology & definitions

From our understanding of symmetry that f^{-1} Is the reflection of f about the line y=x.

f^{-1} is undoing what f does.

Symbols, Units & Equations

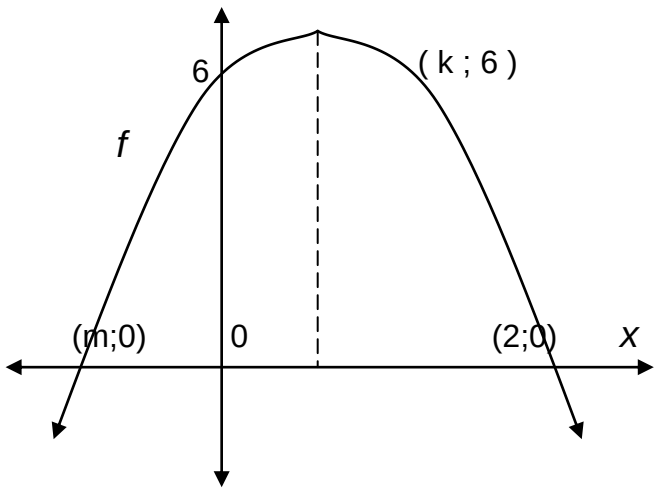
f^{-1} is used to indicate the inverse of f .

X-planation

- **When finding the equation of f^{-1} we simply swap the *x* and *y* values around and work the new equation into y form.**
- **When sketching f^{-1} we take the coordinates of *f* and interchange the *x* and *y* and then plot these new points to get the graph of the inverse.**

X-ample Questions

1. The inverse of a function is $f^{-1}(x) = 2x - 4$. What is the function *f*(*x*)?
   (3)
2. The sketch represents the graph of the parabola *f*, which intersects the x – axis at (m ; 0) and (2 ; 0). It is further given that $\left(\frac{1}{4}; n\right)$ is the turning point of the parabola. Points (0 ; 6) and (k ; 6) also lie on the curve of *f*.



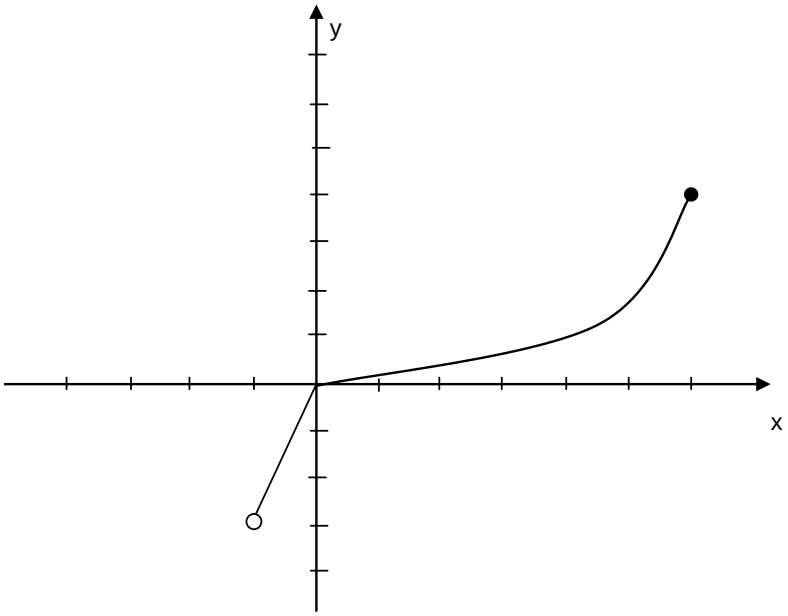
(a) the value of k (1)

(b) the value of m (1)

(c) the value of n, showing all working detail. (5)

(d) the restriction on the range so that the inverse of f is a function. (1)

3. The graph of function g is shown below.

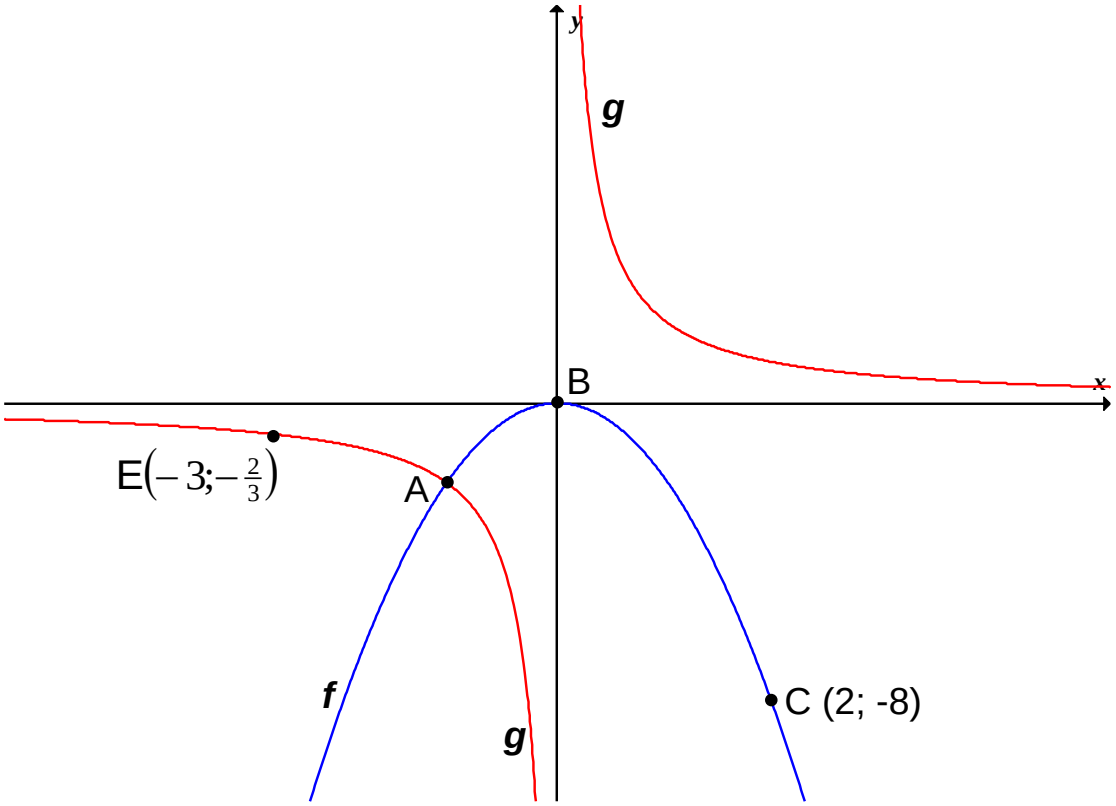


(a) Give the domain and range of the function.
(2)

(b) Draw a neat sketch graph of the inverse function of g.
(3)

(c) Explain why this inverse is a function.
(2)

4. The graphs of g(x) = $\frac{2}{x}$ and f**(x) =** $-2x^2$ are drawn on the same set of axes below.



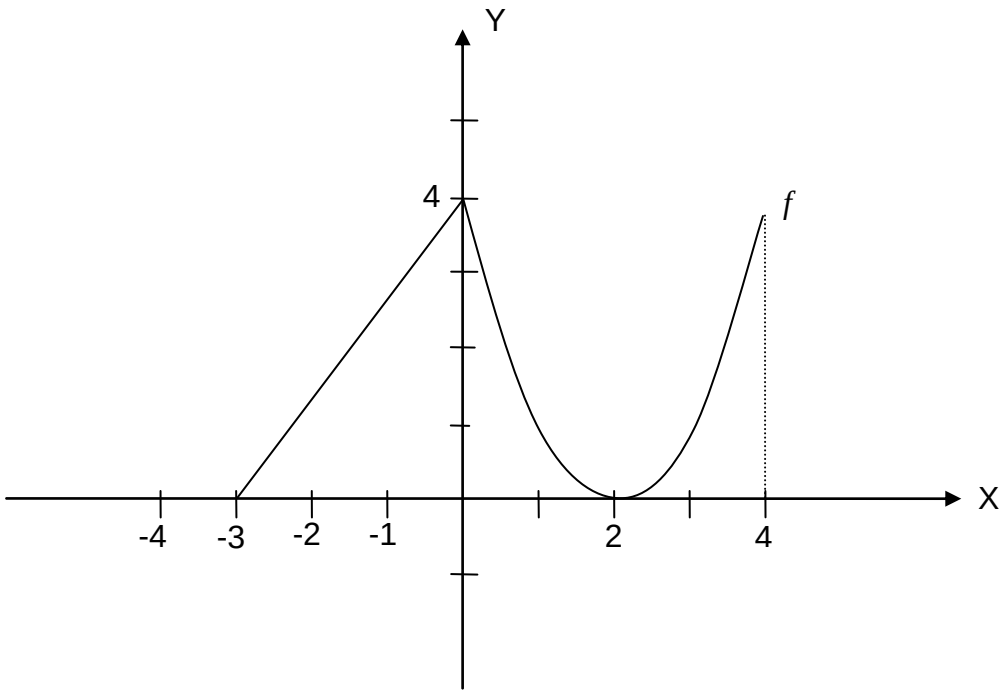
(a) Write the equation of f^{-1} in the form y = …… . (3)

(b) Restrict the domain of f so that f^{-1} is a function. (1)

(c) Write the equation of g^{-1} in the form y=..... (2)

(d) What do you notice about g and g^{-1} (1)

5. Study the sketch below, then answer the questions that follow:



(a) Describe the domain of *f*. (2)

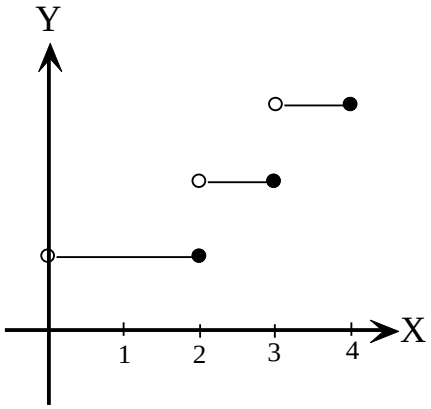
(b) Sketch the graph of the inverse of *f*, on the set of axes provided. (4)

(c) Is f^{-1} a function or a non-function? Explain. (2)

(d) For the straight line *f*, when $x \in [-3;0]$, write down the equation of $f^{-1}(x)$, in the form: $f^{-1}(x) =$ ……. (2)

(e) If the equation of *f* , when $x \in (0;4]$ is: $f(x) = (x-2)^2 + 0$, write down the equation of $g(x)$ which is the reflection of *f* about the x - axis. (2)

6. Use the given sketch of a function *k* to answer the following questions:



1) Why does *k* represent a function? (1)
2) Is *k* one-to-one or a many-to-one function? Explain. (2)
3) What is the range of k^{-1}? (2)
4) Will k^{-1} represent a function? Explain. (2)

7. Sketch the graph of $y = f(x) = 3^x$.

(a) Label at least two defining points clearly. (2)

(b) Determine the inverse of *f* and write it in the form $f^{-1}(x) = ...$ (1)

(c) Sketch the graph of f^{-1} on the same set of axes as *f.* Label at least two defining points clearly. (2)

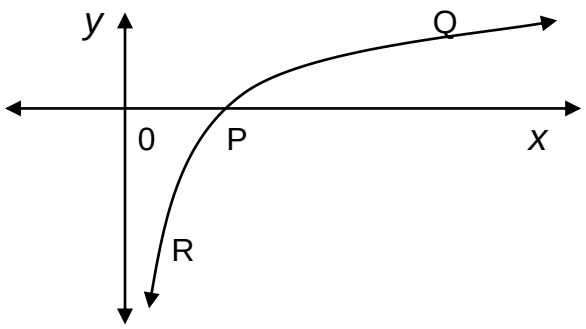
(d) Is f^{-1} a function? Give a reason for your answer. (2)

(e) If $g(x) = f(x - 4) - 3$

(i) Describe the transformation of *f* to *g*. (1)

(ii) Sketch the graph of *g*. Show any new asymptotes and label at least two defining points clearly. (3)

8. The graph is represented by $y = \log_a x$.



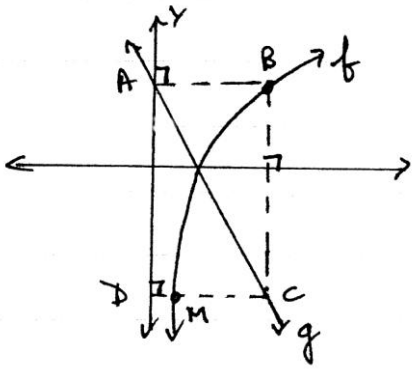
(a) Determine the value of *a* if Q is the point $(8 ; \frac{3}{2})$ (3)

(b) If *a* = 4, determine the value of *b* if R is the point (*b* ; -2) (3)

(c) The inverse of the graph above is translated up by two units and to the left by one unit. Give the equation of the new graph in the form y = … (3)

X-ercise

1.



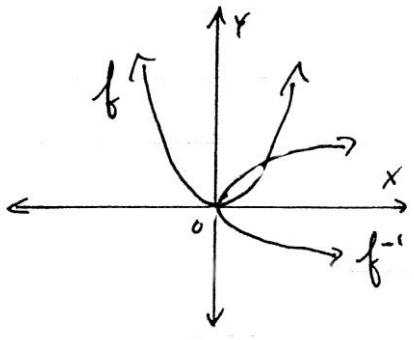
The graphs of $f : y = \log_2 x$ and $g : 2x + y = 2$ are shown on the sketch. A and C are points on g and M and B are points on f.
A is on the y-axis and ABCD is a rectangle.

Determine:

a) the equation of g^{-1}, the inverse of g, in the form y = … (2)

b) the co-ordinates of A and hence the co-ordinates of B. (3)

c) the co-ordinates of M. (3)

2. 

The sketch represents f and f^{-1}

a) f^{-1} is a non-function. Explain this. (1)

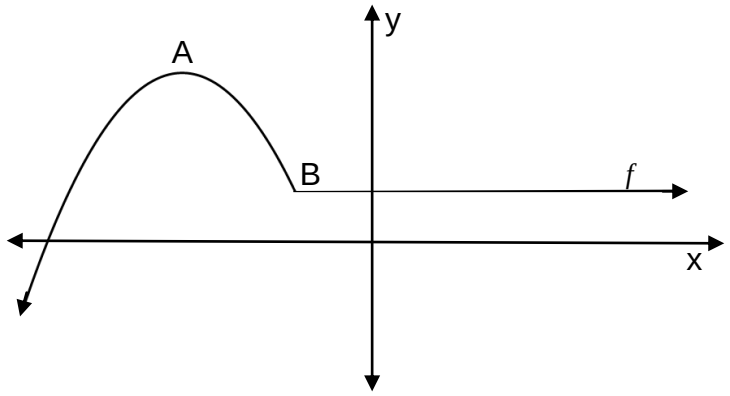
b) Determine a way in which the domain of f should be restricted so that f^{-1} will be a function. (2)

3. If $f(x) = \frac{1}{4}x^2$, give the equation of f^{-1} in the form $y =$ (2)

4. The following functions are given : $f(x) = \log_3 x$ and $g(x) = \frac{1}{x-1}$

(a) Sketch *f(x)* and *g(x)* on the same set of axes. Label all the intercepts with the axes as well as the asymptotes where applicable. (5)
(b) Determine the equation of the inverse of *f(x)* in the form $f^{-1}(x) = \ldots$. (2)
(c) Determine the equation of the inverse of *g(x)* in the form $g^{-1}(x) = \ldots$ (3)

5. Below is a sketch of f. A is a turning point with coordinates $(-5;6)$. B is the point where the parabolic curve and the straight line meet. The coordinates of B are $(-1;2)$.



(a) Is f a function? Explain. (2)
(b) Give the values of x for which f is strictly increasing. (2)
(c) Determine the range of f. (2)
(d) Give one possible restriction on the domain that would ensure that f^{-1} is a function. (2)
(e) Determine $f(3)$. (1)

Answers

1a) $y = -\frac{1}{2}x + 1$

1b) $A\ (0;2)$

$B\ \ (4;2)$

1c) $x = \frac{1}{64}$

2a) Yes it is a function the x values are not repeated

2b) Either $x \leq 0 \quad or \quad x \geq 0$

3 $y = \pm 2\sqrt{x}$

4a) See attached sheet

4b) $f^{-1}(x) = 3^x$

4c) $g^{-1}(x) = \frac{1}{x} + 1$

5a) Yes it is a function the x values do not repeat.

5b) $x \in (-\infty; -5)$

5c) $y \in (-\infty; 6)$

5d) $x \leq -5 \quad or \quad -5 \leq x \leq -1$

5e) $f(3) = 2$