

## Logarithms and Graphs

### Key Concepts

In this session we will focus on summarising what you need to know about:

- Solving basic logarithmic equations using the definition and rules
- Sketching and finding the equations of logarithmic and exponential curves
- Transformations of these graphs.
- Inverses

### Definition

$$\log_a b = c \Leftrightarrow a^c = b \text{ if } a > 0; a \neq 1; b > 0; c \in \mathfrak{R}$$

### Laws

$$\log_m A + \log_m B = \log_m AB \qquad m > 0; m \neq 1; A > 0 \quad B > 0$$

$$\log_m A - \log_m B = \log_m \frac{A}{B}$$

$$n \log_m A = \log_m A^n$$

### Deductions

$$\log_a a = 1 \qquad a > 0; a \neq 1$$

$$\log_a 1 = 0$$

### X-planation

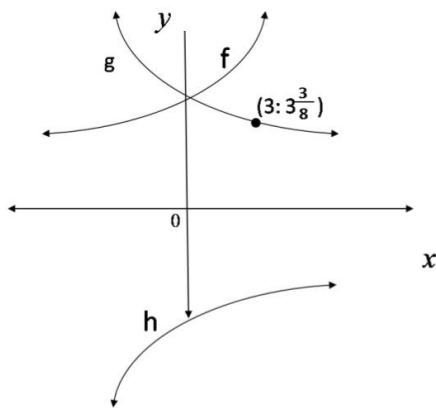
Remember when solving log equations:

- Check restrictions.
- Use rules to condense to one log.
- Sometimes it is better not to work with negative in front of a log so move it over the equal sign.

## X-ample Questions

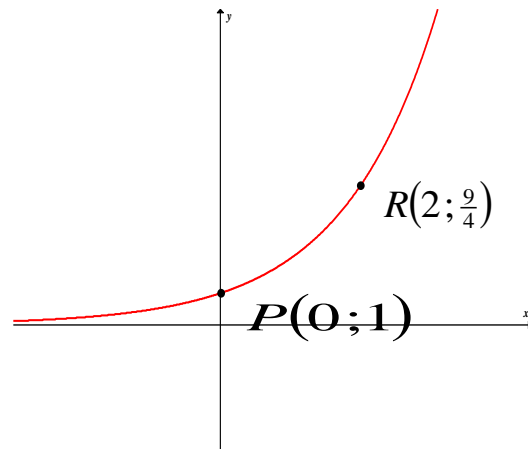
Solve for x

- 1)  $3 \cdot \log_2 (x-1) = 12$
- 2)  $\log_3 8 + \log_3 (x+2) = 2$
- 3)  $\log(2x+1) - \log(x+1) = 1$
- 4)  $\log_5 (x-2) + \log_5 (x+1) = \log_5 4$
- 5)  $\log_2 (x+2) = \log_2 x + 2$
- 6)  $x \log 5 = \log \frac{3}{5} + x \log 3$
- 7)  $45000 = 155000 (1 - 0,18)^x$
- 8) The sketch represents the graphs of the function f with  $f(x) = a^x$  and g which is the reflection of f in the y-axis and h which is the reflection of g in the x-axis.



- a) Calculate the value of  $a$
- b) Write down the equation of  $g$
- c) Write down the equation of  $h$
- d) Write the equation of  $f^{-1}$  making  $y$  the subject of the equation
- e) Sketch the graph of  $f^{-1}$
- f) What is the domain of  $f^{-1}$  ?
- h) If  $h$  is shifted so that its  $y$ -intercept coincides with the origin, what is the equation this new graph?

- 9) Study the curve of the exponential function  $h$ . The function cuts the y-axis at  $P(0; 1)$ , while  $R(2; \frac{9}{4})$  is a point on the curve.



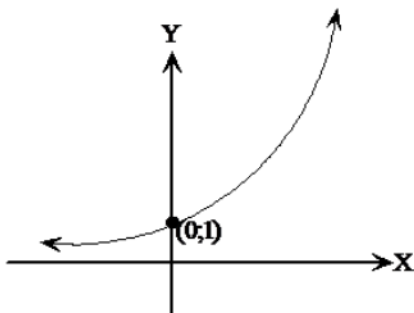
- Determine the equation of  $h$ .
- Write down the equation of  $h^{-1}$  in the form  $y =$
- Sketch the graph of  $h^{-1}$  on the answer sheet provided. Be sure to indicate any intercepts with the axes and the co-ordinates of one other point on the graph

Given that  $f(x) = \left(\frac{1}{3}\right)^x$ ,

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- Determine  $f^{-1}$ , writing your answer in the form,  $f^{-1}(x) = \dots$
- State the domain and range of each of the functions  $f$  and  $f^{-1}$ .
- Sketch the graphs of  $f$  and  $f^{-1}$  on the same system of axes, labelling the graphs clearly.
- Find the value of  $x$  for which  $f(x) = 27$ .

11. Given the graph of  $f(x) = 2^x$  write down the equation of each of the following as transformations in the form  $g(x) =$

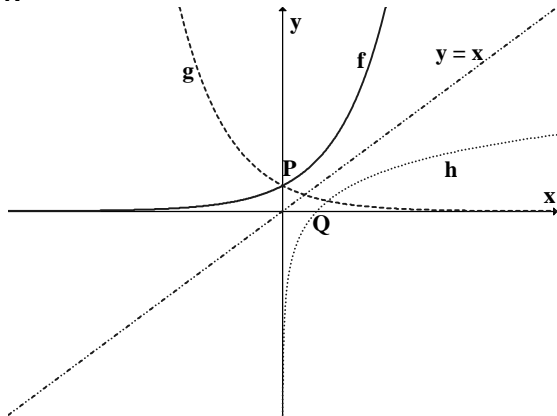


- $f(x)$  is shifted 1 unit downward and 3 units to the right
- the inverse of  $f(x)$
- the reflection of  $f(x)$  in y axis

- 12) If  $\log 2 = a$  and  $\log 3 = b$   
Express in terms of  $a$  and  $b$
- $\log 18$
  - $\log 1,5$
  - $\log 5$

### X-ercise

1.



In the accompanying figure, the graphs of  $f$ ,  $g$ , and  $h$  are given. The graphs of  $f$  and  $g$  are symmetrical with respect to the  $y$  axes, and they intersect in  $P$ . The graph of  $h$  the inverse of  $f$  cuts the  $x$ -axis in  $Q$ .

- If  $f : x \rightarrow a^x$  and  $(1,4) \in f$ , calculate the value of  $a$ . (2)
  - Write down the co-ordinates of  $P$  and  $Q$ , (2)
  - Write down the equations that define  $g$ ,  $h$  and also  $g^{-1}$ . (3)
  - On a sketch the graph of  $g^{-1}$ . (2)
- Solve, for  $x$ :  $\log_x(x-1) + \log_x(x-5) = 2$  (4)
  - Sketch the graph of  $f(x) = \log_{\frac{1}{2}} x$ , on the set of axes provided. (3)
    - Describe the domain of  $f$ . (2)
    - Sketch, on the same set of axes,  $g$ , the reflection of  $f$  about the  $x$ -axis, and write down the equation of  $g$  in the form  $g(x) = \dots\dots\dots$  (3)
  - Solve for  $x$ :  $\log_3(2x-5) + \log_3 x = 1$  (5)
  - Solve for  $x$ :  $2 \log_3(2x-5) = 4$  (3)
  - Solve for  $x$ :  $76 = 3^{x+1}$  (4)

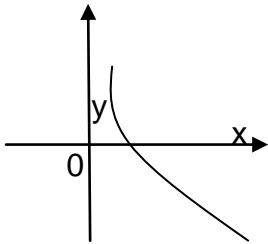
## Answers

- 1 a) 4  
b) P(0;1)  
Q(1;0)

$$g(x) = 4^{-x}$$

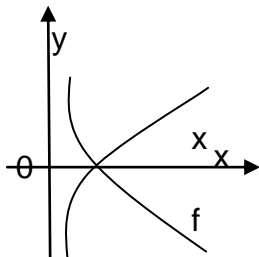
c)  $h(x) = \log_4 x$

$$g^{-1}(x) = \log_{\frac{1}{4}} x$$



2.  $x = \frac{5}{6}$

3.



4.  $x=3$   
5.  $x=7$   
6.  $x=2,94$