Functions & Graphs: Logs & Inverses 22 APRIL 2013

Lesson Description

In this lesson, we:

- Revise exponential functions from grade 11
- Define the inverse of an exponential function
- Look at exam type questions.

Key Concepts

Terminology

- A function is a mathematical rule that maps an input value to a unique output value.
- The domain of a function is the set of all input values
- The range of a function is the set of all output values

Vertical Line Test

We use a ruler to perform the "vertical line test" on a graph to see whether it is a function or not. Hold a clear plastic ruler parallel to the y-axis, i.e. vertical.

Move it from left to right over the axes.

If the ruler only ever cuts the curve in one place only throughout the movement from left to right, then the graph is a function.

If the ruler ever passes through two or more points on the graph, the graph will not be a function but a relation.

Inverse of a function

An inverse of a function is a mapping of all the output values to the input values. The inverse of a function may not be a function. It is a reflection about the line y = x

Sketching graphs

Rules for sketching exponential graphs of the form $y = ab^{x+p} + q$

1. Write down the **horizontal asymptote** and draw it on a set of axes:
   Horizontal asymptote: $y = q$
2. Plot **three graph points** on your set of axes.
3. Draw the newly formed graph.

Logarithmic functions

The inverse of the exponential function is called the logarithmic function.

Consider the function $y = a^x$. The inverse of this graph is $x = a^y$

It is now possible to make *y* the subject of the formula in the equation $x = a^y$ by means of the concept of a logarithm.

If $x = a^y$, then it is clear from the definition of a logarithm that $\log_a x = y$. In other words, we can write the inverse of the function $f(x) = a^x$ as $f^{-1}(x) = \log_a x$.

Questions

Question 1

Given that $f(x) = \left(\frac{1}{3}\right)^x$

a. Determine f^{-1} writing your answer in the form $f^{-1}(x) = ...$

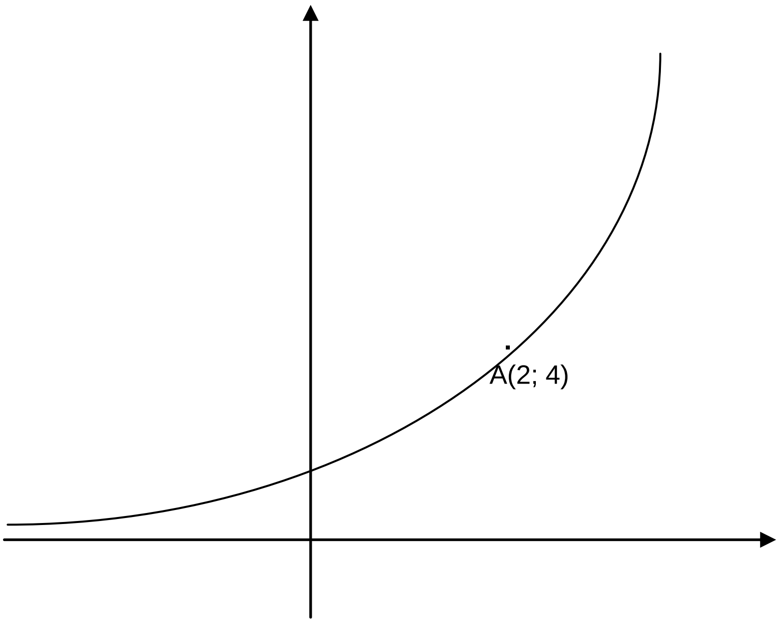
b. State the domain and range of f^{-1}.

c. Give the equation of the line about which f and f^{-1} are symmetric.

d. Determine *g*, the reflection of f in the *y* axis.

Question 2

In the 2010 FIFA logo there is a curve that approximates the exponential curve.



f(x) = a^x ; x Є [–2; 3]. A(2; 4) is a point on f.

a.) Find the value of a.
b.) Find the equation of g if g is a reflection of f about y=x.
c.) Write your answer in the form : g(x) = …….. ; x Є [….]
d.) g is reflected about the x-axis to obtain h,
   i) Write down the equation of h in the form h(x) = ...
   ii) Find the value of h(1)
   iii) Find the value of x if h(x) = 2

Question 3

Consider the functions $f(x) = 2x^2$ *and* $g(x) = \left(\frac{1}{2}\right)^x$

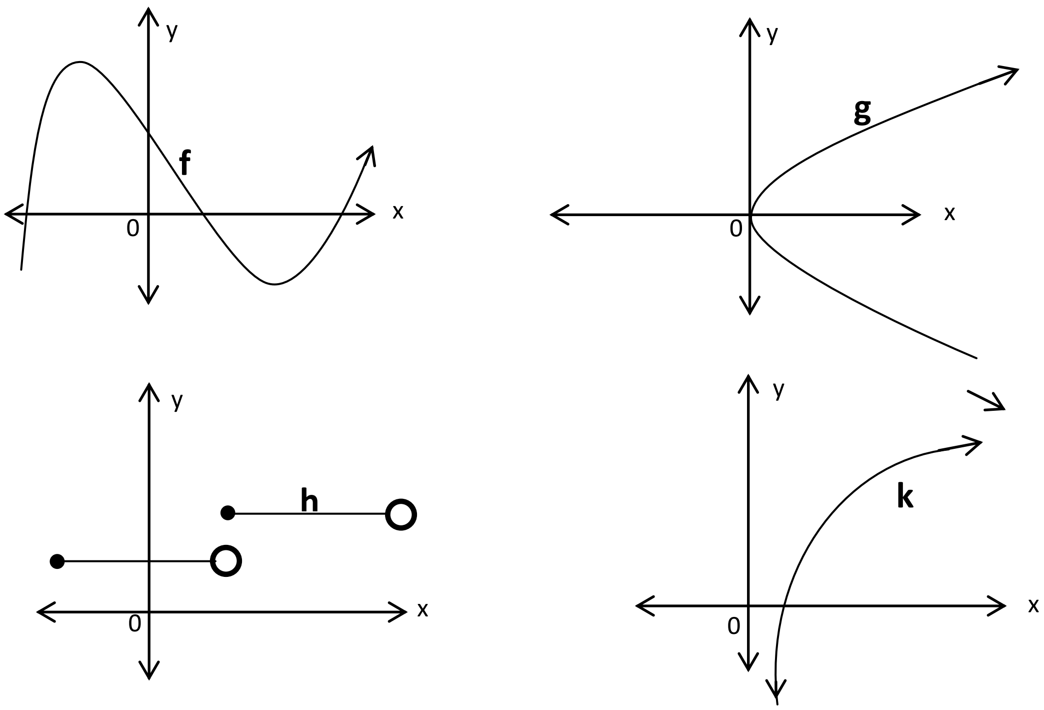
Restrict the domain of f so that the inverse of f will also be a function.

Write the inverse of g in the form $g^{-1}(x) = \ldots$

The inverse of a function is $f^{-1}(x) = 2x - 4$. Write down the equation of $f(x)$?

Question 4

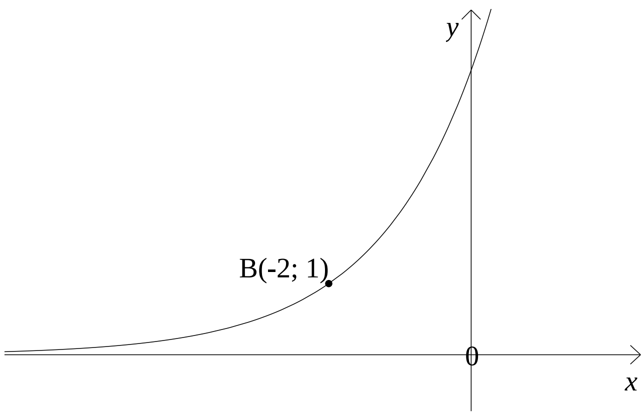
Given the four sketch graphs below, (not drawn to scale), answer the following questions:



State whether the following are TRUE or FALSE:

a.) f is a many-to-one function.
b.) The inverse of g is a function.
c.) h is not a function.
d.) k is a many-to-one function.
e.) The inverse of f is a function.
f.) g is not a function.
g.) h is a one-to-one function.
h.) The inverse of k is not a function.

Question 5



a.) Write down the asymptote of $h(x)$. (1)
b.) Determine $h^{-1}(x)$. (3)