

SESSION 2: RATIO, PROPORTION, RATES AND PERCENTAGES

KEY CONCEPTS:

- Ratio
- Proportion
- Rates
- Percentages

X-PLANATION

1. Ratio:

A ratio is a comparison of two numbers (called terms of the ratio).

Ratios have no units since the quantities being compared are of the same kind or type.

Ratios can be written in different ways:

- In words → a to b
- With a colon → a:b
- As a fraction → $\frac{a}{b}$

Example:

Suppose there are 12 boys and 9 girls in a class. The ratio of boys to girls can be written

- In words → 12 to 9
- With a colon → 12:9
- As a fraction $\frac{12}{9}$

Ratios can be written in equivalent form and therefore used for comparison.

2. Proportion

When two ratios are equal, the four quantities are said to form a proportion.

Example:

1. $3/12 = 6/24$

2. You want to mix cement to patch a crack in the wall and have noticed that the builder mixes 6 pockets of cement with 18 pockets of sand. If you decide to mix 2 cups of cement with 6 cups of sand, you are using the cement and the sand in the same proportion as the builder.

Direct Proportion

When two quantities are in direct proportion as the one increases or decreases the other increases or decreases by the same proportion.

Example

Cost of petrol and number of litres are in direct proportion. If you pay R24 for two litre, you will pay five times the cost (R120) for five times the number of litres (two x 5 = 10 litres). If you only want to pay R60 (half the price) you will only get half the number of litres (half of 10 = 5 litres).

Inverse Proportion

When two quantities are in inverse proportion as the one increases the other decreases by the same proportion or as the one decreases the other increases by the same proportion.

Example

The table of values below are in inverse proportion:

Amount	Cost
1	R100
2	R 50
4	R 25

3. Rates

A rate is a special type of ratio. For rates we compare two different quantities.

Examples

The cost of petrol per litre: R 12 per litre
 Speed: Distance travelled per hour: 60km/h
 Tax Rate: VAT is 14% of cost of goods or services (constant rate)

4. PERCENTAGES:

A percentage is a portion of a whole, where the whole is one hundred. Every percentage is then a fraction out of 100 (the whole). It is for this reason that we write a percentage as a fraction with a denominator of 100.

E.g. 40% is shorthand for $\frac{40}{100}$ or 0,40

Percentage has been adopted quite comfortably into day to day language because:

- People find it easier to visualize / comprehend percentage than actual amounts. For example one would have a better sense of how popular a candidate was if you heard "Karen got 70% of the votes" compared with "Karen got 4 389 of the 6 270 votes cast".
- It makes comparisons easier. For example, people find it easier to make sense of the statement: "37,5% of the population got ill this year in comparison with 44,4% last year" than they would the statement: " $\frac{3}{8}$ of the population got ill this year in comparison with $\frac{4}{9}$ last year"

When dealing with percentage, below are five different types of questions you may be asked.

- a) If given an amount to find out how much of the total the amount is in %:
- i) Thandi gets 20 out of 25 for her Test. How much is the percentage of the total?

$$\frac{20}{25} \times \frac{100}{1} = 80\%$$

- ii) Work out the percentage of $\frac{2}{5}$

$$\text{this will be } 2 \div 5 = 0.4 \times 100 = 40\%$$

- b) If given the percentage to find out the new total:

- i) An article cost R15 and VAT is 5%.

We would work out the amount due as follows:

$$\begin{aligned} R15 \times 5\% &= 0,75 \\ R15 + 0,75 &= R15,75 \\ &\text{or} \\ R15 \times 105\% &= R15,75 \\ (100\% + 5\%) \end{aligned}$$

- ii) I exchange R1 250 in foreign exchange and then pay a 12% commission fee. How much in total do I pay to the cashier?

$$\begin{aligned} R1\ 250 \times 12\% &= R150 \\ R1\ 250 + R150 &= R1\ 400 \\ &\text{or} \\ R1\ 250 \times 112\% &= R1\ 400 \\ (100\% + 12\%) \end{aligned}$$

- iii) The butcher increased all his prices by 8%. If mince was R21,99 per kg, what would you pay now for 1 kg?

$$\begin{aligned} R21,99 \times 8\% &= R1,76 \\ R21,99 + R1,76 &= R23,75 \\ &\text{or} \\ R21,99 \times 108\% &= R23,75 \\ (100\% + 8\%) \end{aligned}$$

c) If given the new amount to find out the original amount:

i) If the price of an article after 5% VAT is added is R15,75, what is the cost excluding VAT?

$$R15,75 \div 105\% = R15$$

ii) If I had R1 400 and went to exchange money but had to pay 12% commission, how much money could I exchange?

$$R1400 \div 112\% = R1\ 250$$

iii) After a drastic price increase of 8%, I pay R23,75 for 1kg mince. How much did the mince previously cost?

$$R23,75 \div 108\% = R21,99$$

d) If given two amounts to find the % increase or decrease:

$$\frac{\text{New amount} - \text{initial amount}}{\text{Initial Amount}} \times \frac{100}{1}$$

e) If given the percentage to convert into a common fraction:

Convert 25% to a common fraction.

$$\text{Key 25 into calculator and } \div 100 = \frac{1}{4}$$

X-AMPLE QUESTIONS:

Question 1:

The following recipe serves 10 people; Mpho would like to serve 15 people.

Pancakes
250g cake flour
2 eggs
500ml water
5ml salt

- a) How many eggs does she need? (3)
- b) How much cake flour does she need? (2)
- c) If 250ml of water is equal to one cup how many cups does she need for 15 people? (4)

Question 2:

- a) One bag of dog food is 8kg. If 2 dogs eat 450g of dog food each a day. How many bags of dog food do we need for 30 days? (6)
- b) If one 8kg bag of dog food costs R44,99. How much will it cost to feed the two dogs for 30 days? (2)

Question 3:

- a) Sipho is going shopping he sees that mince meat costs R24,99 per kg how much will he pay for 250g? (3)
- b) How much mince can he buy for R76,53? (2)
- c) Sipho also needs to buy tea: Tea bags come in four different sized boxes: 62,5g for R5,39; 125g for R14,49; 250g for R19,49 and 500g for R36,89.
- i) Which size box is the best buy? (9)
- ii) Which size box is the worst buy? (1)

Question 4:

Which is the better buy?

- a) 100 Trinco teabags @ R14,95
- b) 80 Freshpack teabags @ R11,99 (5)

Question 5:

A family, earning R3 000 per month, spends approximately R1 630 per month on food.

- a) The mother of the family. Mrs Kay goes shopping for food every Saturday. If she is to keep within the food budget what is the maximum amount she can spend each week, to the nearest R100? (2)
- b) She needs to buy the following basic items every week
- 9 litres of milk @ R4,98 per litre
 - 7 loaves of bread @ R4,70 each
 - 2kg rice @ R3,98 per kg
- What is the total for her basic purchases? (3)

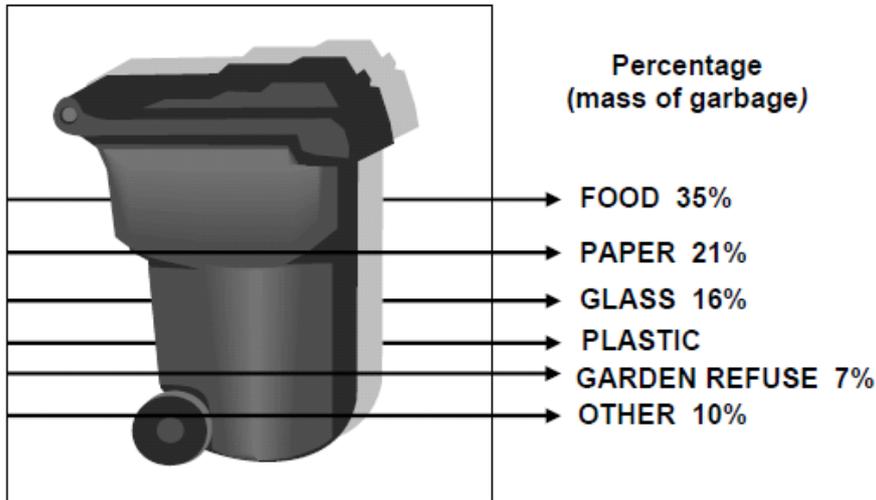
Question 6:

Calculate the following:

- a) 36 out of 40 as a percentage (2)
- b) 30% of R42,90 (2)
- c) R340 decreased by 4% (3)
- d) 28 expressed as a percentage of 84 (3)
- e) A loaf of bread costs R9,22. Last year the same loaf cost R7,58. What is the percentage increase? (3)
- f) An article costing R31,92 includes VAT of 14%. What was the original price of the article before the VAT was added? (3)

Question 7

The diagram below represents the percentage composition of the garbage disposed of by the Smith family in a week. The average full garbage bin has a mass of 35 kg, which excludes the mass of the bin.



- Calculate the percentage of plastic waste that is disposed by the Smith family per week. (2)
- Calculate the mass of food in kilograms that the Smith family wastes per week. (2)
- The local municipality will recycle glass and paper. Determine the mass of the remaining garbage (excluding glass and paper). (3)