ANALYTICAL GEOMETRY

Checklist

Make sure you:

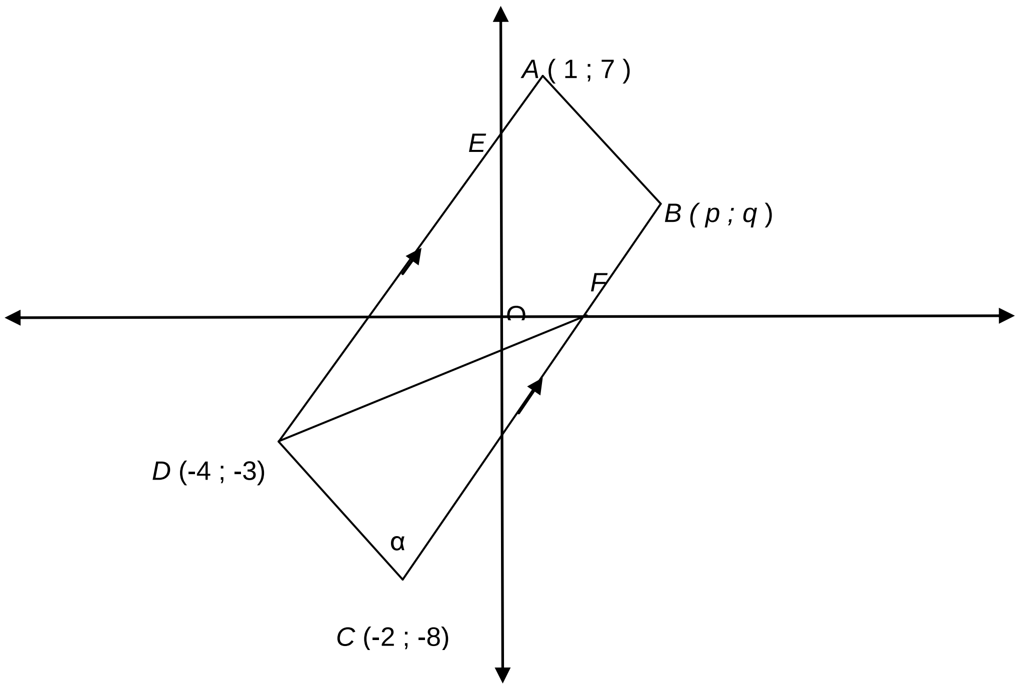
- Can **recognise** and **apply** the **distance formula.**
- Can **recognise** and **apply** the **midpoint formula**.
- Can calculate the **gradient** or **slope** of a straight line.
- Know the relationships between **parallel** and **perpendicular** lines.
- Can calculate the **inclination** of a straight line.
- Can find the equation of a circle with centre **the origin** and **centre (a ; b).**
- Can recall the properties of quadrilaterals and circle theorems, especially the relationships of tangents to a circle.

Exam Questions

Question 1

(Adapted from Feb 2013, Paper 2, Question 4)

In the diagram below, trapezium $ABCD$ with AD // BC is drawn. The coordinates of the vertices are $A(1\,;7), B(p\,;q), C(-2\,;\,-8)$ and $D(-4\,;\,-3)$. BC intersects the x - axis at F. $D\hat{C}B = \alpha$.



1.1. Calculate the gradient of *AD*. (2)

1.2. Determine the equation of *BC* in the form $y = mx + c$ (3)

1.3. Determine the coordinates of point *F.* (2)

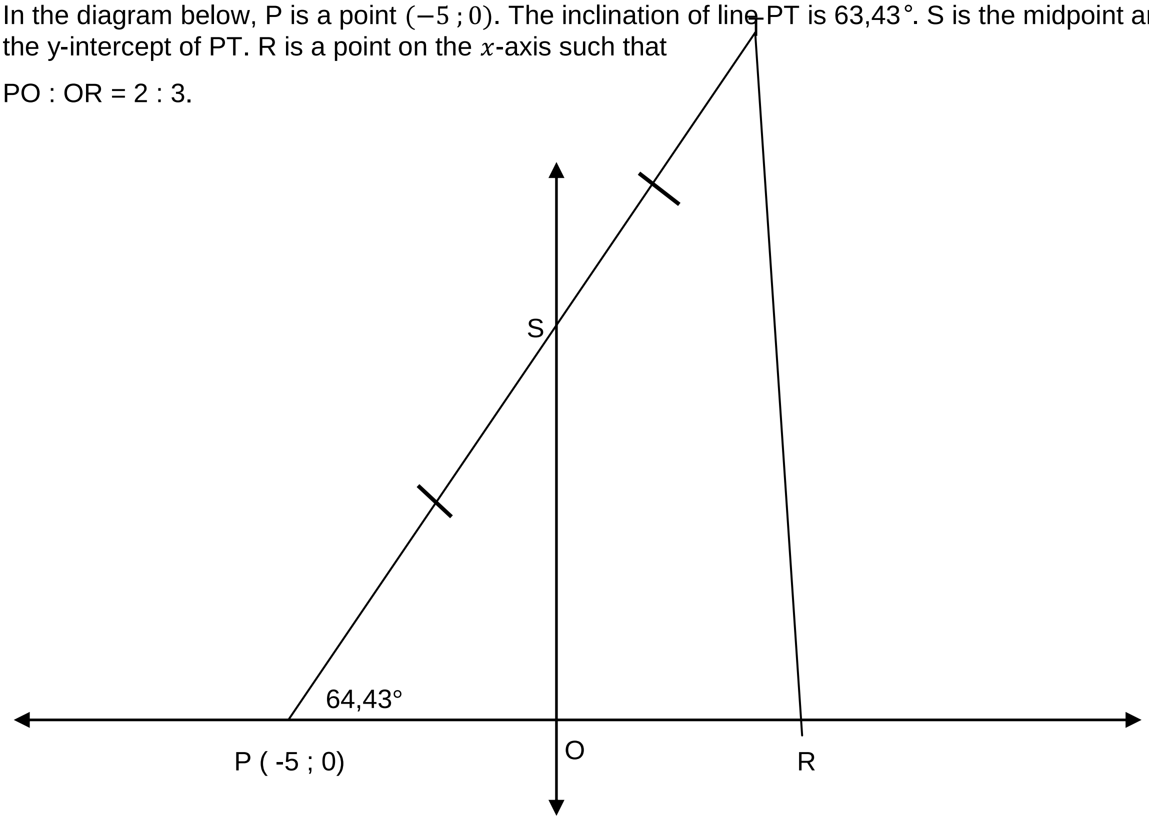
1.4. Show that $\alpha = 48{,}37°$. (4)

Question 2

(Adapted from Nov 2013, Paper 2, Question 5)

In the diagram below, P is a point $(-5\,;0)$. The inclination of line PT is 63,43°. S is the midpoint and the y-intercept of PT. R is a point on the x-axis such that

PO : OR = 2 : 3.



2.1. Determine

2.1.1. The gradient of PT, correct to the nearest integer value. (2)

2.1.2. The equation of PT in the form $y = mx + c$. (2)

2.1.3. The distance PS in surd form. (3)

2.1.4. The coordinates of T. (2)

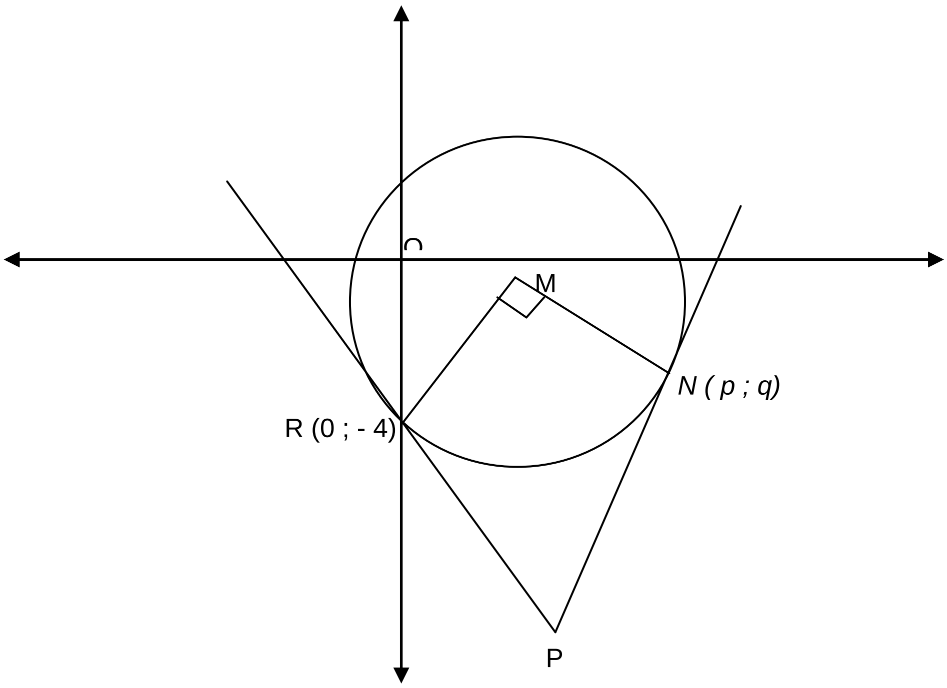
2.2. Determine the coordinates of R. (2)

2.3. Calculate the area of ΔPTR. (4)

Question 3

(Adapted from Nov 2013, Paper 2, Question 6)

In the diagram below, M is the centre of the circle having the equation $x^2 + y^2 - 6x + 2y - 8 = 0$. The circle passes through R(0 ; -4) and N(*p* ; *q*). $R\hat{M}N = 90°$. The tangents drawn to the circle at R and N meet at P.

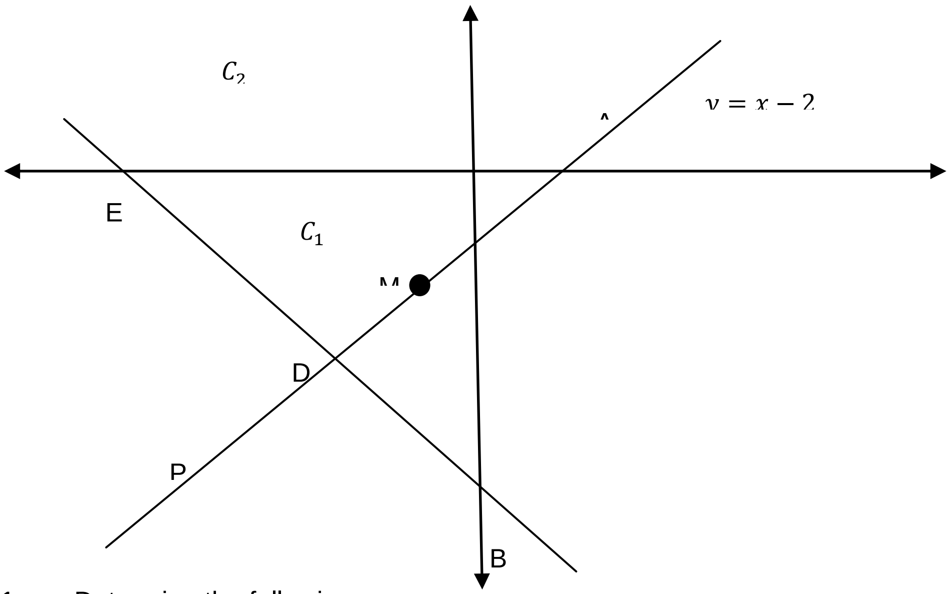


3.1.	Show that M is the point (3 ; -1).	(4)
3.2.	Determine the equation of MR in the form $y = mx + c$.	(3)
3.3.	Show that $q = 2 - p$.	(4)
3.4.	Determine the values of *p* and *q*.	(5)
3.5.	Determine the equation of the circle having centre O and passing through N.	(2)
3.6.	Calculate the area of the circle centred at M.	(2)
3.7.	Calculate the ratio in its simplest form: $\frac{NP}{MP}$.	(4)

Question 4

(Adapted from Feb 2013, Paper 2, Question 5)

Circles C_1 and C_2 in the figure below have the same centre M. P is a point on C_2. PM intersects C_1 at D. The tangent DB to C_1 intersects C_2 at B. The equation of circle C_1 is given by $x^2 + 2x + y^2 + 6y + 2 = 0$ and the equation of line PM is $y = x - 2$.



4.1. Determine the following:

4.1.1. The coordinates of centre M. (3)

4.1.2. The radius of circle C_1. (1)

4.2. Determine the coordinates of D, the point where line MP and circle C_1 intersect. (5)

4.3. If it is given that DB = $4\sqrt{2}$, determine MB, the radius of circle C_2. (3)

4.4. Write down the equation of C_2 in the form $(x - a)^2 + (y - b)^2 = r^2$ (2)

4.5. Is the point F $(2\sqrt{5} ; 0)$ inside circle C_2? Support your answer with calculations. (4)

Answers

Exam Questions

Question 1

1.1 $m_{AD} = \frac{y_A - y_D}{x_A - x_D}$

$$m_{AD} = \frac{7-(-1)}{1-(-4)}$$
$$m_{AD} = \frac{10}{5}$$
$$m_{AD} = 2$$

1.2 $y = mx + c$

$m = 2$
Substitute $(-2;-8)$:
$-8 = 2(-2) + C$
$-8 = -4 + C$
$-4 = c$
$\therefore y = 2x - 4$

1.3 $x - int$ let $y = 0$

$0 = 2x - 4$
$4 = 2x$
$x = 2$
$\therefore F(2;0)$

1.4 $tan\ \theta = m$

$\tan\theta = 2$
$\theta = tan^{-1} 2$
$\theta = 63,43$

$$m_{CD} = \frac{-8-(-3)}{-2-(-4)}$$
$$m_{CD} = -\frac{5}{2}$$

$$\tan\beta = -\frac{5}{2}$$

$\beta = 180° - 68,2°$

$\beta = 111.8°$

$\alpha = 111,8° - 63,43°$

$\alpha = 48,37°$

Question 2

2.1.1 $\mathrm{m} = \tan 64{,}43°$

$\mathrm{m} = 2{,}089968596$
$m \approx 2$

2.1.2 $y = 2x + c$

Substitute $(-5; 0)$
$0 = 2(-5) + c$
$c = 10$
$y = 2x + 10$

2.1.3 $PS^2 = 5^2 + 10^2$

$PS^2 = 125$
$PS = 5\sqrt{5}$

2.1.4 $\frac{-5+x}{2} = 0$

$x = 5$

$$\frac{0 + y}{2} = 10$$
$y = 20$

$\therefore (5; 20)$

2.2 $\frac{PO}{OR} = \frac{2}{3}$

$$\frac{5}{OR} = \frac{2}{3}$$
$2OR = 15$
$$OR = \frac{15}{2}$$
$$\therefore R\left(\frac{15}{2}; 0\right)$$

2.3 $Area = \frac{1}{2}\left(5 + \frac{15}{2}\right)(20)$

$Area = 125 units^2$

Question 3

3.1 Show that M is the point (3 ; -1). (4)

$$x^2 - 6x + 9 + y^2 + 2y + 1 = 8 + 9 + 1$$
$$(x-3)^2 + (y+1)^2 = 18$$
$$\therefore M(3; -1)$$

3.2 Determine the equation of MR in the form $y = mx + c$. (3)

$$y = mx + c$$
$$m = \frac{-1-(-4)}{3-0}$$
$$m = \frac{3}{3} = 1$$
$$\therefore y = x + c$$
$$substitute\ (0; -4)$$
$$-4 = 0 + c$$
$$c = -4$$
$$\therefore y = x - 4$$

3.3 Show that $q = 2 - p$. (4)

$$m_{MN} = -1$$
$$-1 = \frac{q+1}{p-3}$$
$$q + 1 = -p + 3$$
$$q = 2 - p$$

3.4 Determine the values of *p* and *q*. (5)

$$MR = MN$$
$$\sqrt{(3-0)^2 + (-1-(-4))^2} = \sqrt{(p-3)^2 + (q+1)^2}$$
$$\sqrt{3^2 + 3^2} = \sqrt{(p-3)^2 + (q+1)^2}$$
$$\sqrt{18} = \sqrt{(p-3)^2 + (2-p+1)^2}$$
$$18 = (p-3)^2 + (3-p)^2$$
$$18 = p^2 - 6p + 9 + 9 - 6p + p^2$$
$$2p^2 - 12p = 0$$
$$2p(p-6) = 0$$
$$2p = 0 \text{ or } p - 6 = 0$$
$$\therefore p = 0 \text{ or } p = 6$$

$$\therefore q = 2 - 6$$
$$q = -4$$

3.5 Determine the equation of the circle having centre O and passing through N.

$$x^2 + y^2 = r^2$$
$$(6)^2 + (-4)^2 = r^2$$
$$36 + 16 = r^2$$
$$r^2 = 52$$
$$\therefore x^2 + y^2 = 52$$

3.6 Calculate the area of the circle centred at M. (2)

$$A = \pi r^2$$
$$A = \pi\left(\sqrt{18}\right)^2$$
$$A = 18\pi$$

3.7 Calculate the ratio in its simplest form: $\frac{NP}{MP}$. (4)

$$\frac{NP}{MP} = \sin 45°$$
$$\frac{NP}{MP} = \frac{1}{\sqrt{2}}$$

Question 4

4.1.1 The coordinates of centre M. (3)

$$x^2 + 2x + 1 + y^2 + 6y + 9 = -2 + 1 + 9$$
$$(x + 1)^2 + (y + 3)^2 = 8$$
$$\therefore M(-1; -3)$$

4.1.2 The radius of circle C_1. (1)

$$r^2 = 8$$
$$\therefore r = \sqrt{8}$$

4.2 Determine the coordinates of D, the point where line MP and circle C_1intersect. (5)

$(x+1)^2+(y+3)^2=8$
$(x+1)^2+(x-2+3)^2=8$
$(x+1)^2+(x+1)^2=8$
$2(x+1)^2-8=0$
$2(x^2+2x+1)-8=0$
$2x^2+4x+2-8=0$
$2x^2+4x-6=0$
$x^2+2x-3=0$
$(x-1)(x+3)=0$
$x\neq 1$ or $x=-3$

Substitute $x=-3$ in $y=x-2$
$y=-3-2$
$y=-5$

$\therefore D(-3;-5)$

4.3 If it is given that DB = $4\sqrt{2}$, determine MB, the radius of circle C_2. (3)

$MB^2=MD^2+BD^2$
$MB^2=\left(\sqrt{8}\right)^2+\left(4\sqrt{2}\right)^2$
$MB^2=8+16(2)$
$MB^2=40$
$MB=\sqrt{40}$
$MB=2\sqrt{10}$

4.4 Write down the equation of C_2 in the form $(x-a)^2+(y-b)^2=r^2$ (2)

$(x+1)^2+(y+3)^2=40$

4.5 Is the point F ($2\sqrt{5}$; 0) inside circle C_2? Support your answer with calculations. (4)

Substitute $F\left(2\sqrt{5};0\right)$ into $(x+1)^2+(y+3)^2=40$
If $r^2<40,$ then F is inside C_2.
$\left(2\sqrt{5}+1\right)^2+(0+3)^2$
$=4(5)+4\sqrt{5}+1+9$
$=30+4\sqrt{5}<40$
$\therefore F$ lies inside C_2