EUCLIDEAN GEOMETRY: SIMILARITY

Checklist

Make sure you learn proofs of the following theorems:

- A line drawn parallel to one side of a triangle divides the other two sides proportionally
- equiangular triangles are similar

Remember to use correct reasoning when using theorems to state your case:

THEOREM STATEMENT	ACCEPTABLE REASONS
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side	Midpt Theorem
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt || to 2^{nd} side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line || one side of Δ **OR** prop theorem; name || lines
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| Δ's **OR** equiangular Δs
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	Sides of Δ in prop

Knowledge of geometry from previous grades will be integrated into questions in the exam. Also remember that circle geometry questions will be combined with similarity of triangles.

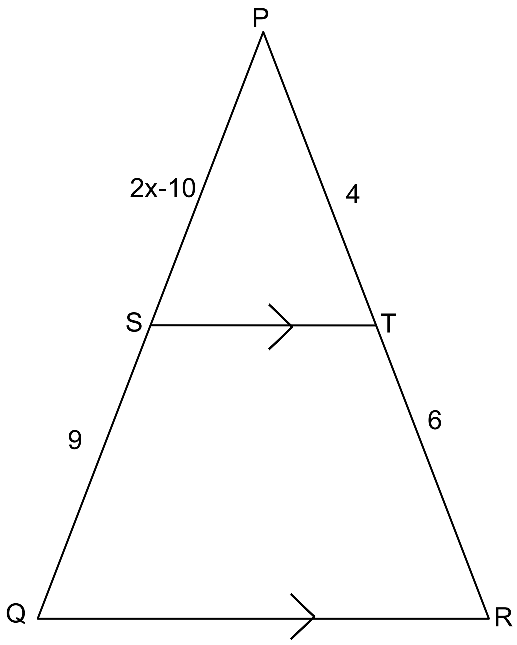
NB:
If you have attempted to answer a question more than once, make sure you cross out the answer you do not want marked, otherwise your first answer will be marked and the rest ignored.

Exam Questions:

Question 1

In the diagram below, ΔPQR has S on PQ and T on PR such that ST || QR. PT = 4 units, SQ = 9 units, TR = 6 units and PS = 2x – 10 units. Calculate the value of PS.

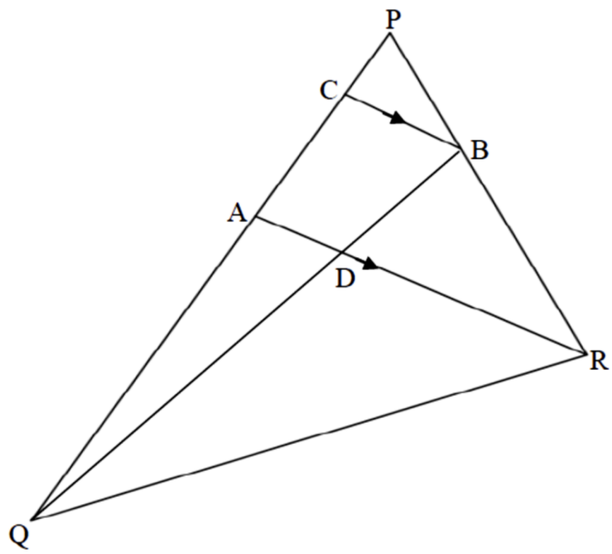
(3)



Question 2

(Adapted from Feb/March2014, Paper 3, Question 9)

In ΔPQR below, B lies on PR such that 2PB = BR. A lies on PQ such that PA : PQ = 3 : 8 BC is drawn parallel to AR.



2.1 Write down the value of $\frac{Area\ of\ \Delta PRA}{Area\ of\ \Delta\ QRA}$ (2)

2.2 Calculate the ratio of $\frac{BD}{BQ}$. Show all working. (5)

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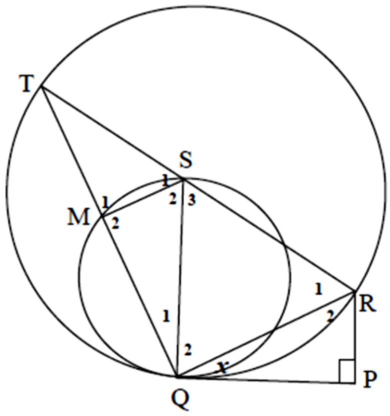
Question 3

(Adapted from Nov 2013, Paper 3, Question 12)

PQ is a common tangent to the two circles. S is the centre of the larger circle and QS is a diameter of the smaller circle. T and R are points on the larger circle and TSR is a straight line. TQ cuts the smaller circle at M and RP⊥QP.



Prove giving reasons:

3.1 TM = MQ (2)

3.2 QR bisects $\angle PRS$ (2)

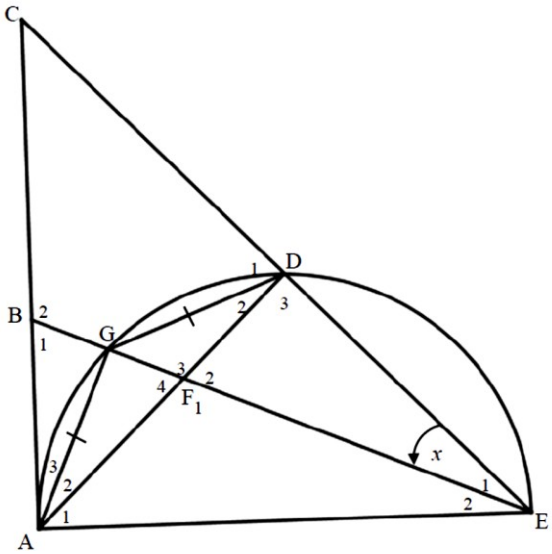
3.3 ΔPQR ||| ΔQTR (3)

3.4 $RQ^2 = 2SQ.RP$ (3)

Question 4

(Adapted from Exemplar 2014, Paper 2, Question 9)

In the figure AGDE is a semicircle. AC is the tangent to the semicircle at A and EG produced intersects AC at B. AD intersects BE in F. AG = GD and $E_1 = x$



4.1 Write down, with reasons, FOUR other angles each equal to x. (8)

4.2 Prove that BE.DE = AE.FE (7)

4.3 Prove that $\angle B_1 = \angle D_1$ (4)

SOLUTIONS TO EUCLIDEAN GEOMETRY: SIMILARITY

Question 1

1. $\frac{PS}{SQ} = \frac{PT}{TR}$ (prop thrm, ST II QR) (1)

$\frac{2x-10}{9} = \frac{4}{6}$

$\therefore x = 8$ (1)

∴ PS=6 (1)

Question 2

2.1 $\frac{Area\ of\ \Delta PRA}{Area\ of\ \Delta QRA} = \frac{PA}{QA} = \frac{3}{5}$ (equal altitudes) (2)

2.2 $\frac{BD}{BQ} = \frac{CA}{CQ}$ $(prop\ theorem)$ (1)

$\frac{PB}{BR} = \frac{1}{2}$ $(given\ 2PB = BR)$ (1)

$\therefore PC: CA = 1:2$ $(prop\ thrm)$ (1)

Let PC=y ,AC= 2y and CQ=5y PA:AQ=3:8 (1)

CA:CQ= 2:7

$\therefore \frac{BD}{BQ} = \frac{2}{7}$ (1)

Question 3

3.1 ∠M= 90 (angle in semi-circle) (1)

$\therefore SM \perp TQ$

$\therefore TM = MQ$ $(line\ from\ centre\ theorem)$ (1)

3.2 $\angle R_2 = 90 - x$ $(sum\ of\ \angle'sof\ \Delta QRP)$

$\angle PQR = \angle T = x$ $(\tan chord\ theorem)$

$\angle R_1 = 90 - x$ $(sum\ of\ \angle'sof\ \Delta QTR)$ (3)

QR bisects ∠PRS

3.3) $in\ \Delta PQR\ and\ \Delta QTR$

i) ∠PQR=∠T =x (tan chord thrm)

ii) ∠R1=∠R2 proved

iii) ∠P=∠TQR (remaining angle)

$\therefore \Delta PQR\ III\ \Delta QTR$ (a,a,a) (3)

3.4 $\frac{PR}{RQ} = \frac{RQ}{TR}$ $\Delta PQR \; III \; \Delta QTR$

$RQ^2 = PR.TR$

$TR = 2SR = 2SQ \quad (SR = SQ, radii) \therefore RQ^2 = 2SQ.RP$ (3)

Question 4

4.1 $\angle A_2 = \angle E_1 = x$ $(<' s \text{ in } same\ segment)$ (2)

$E_2 = x$ $(equal\ chords\ GD\ \& AG)$ (2)

$D_2 = E_2 = x$ $(<' s\ in\ same\ segment)$ (2)

$\angle A_3 = D_2 = x$ $(\tan chord\ thrm)$

$\therefore \angle A_3 = x$ (2)

4.2 $\frac{BE}{AE} = \frac{FE}{DE}$ $(rewriting\ original)$

$in\ \Delta BEA\ \&\ \Delta FED$

i) ∠E1=∠E2 (proved) (1)

ii) ∠A =90º (diameter ⊥ to tangent)

∠D3 =90º (< in semi circle)

∠A=∠D3 (3)

iii) B1 =F2 (remaining angle)

$\therefore\ \Delta BEA\ III\ \Delta FED$ (<,<,<) (1)

$\therefore \frac{BE}{AE} = \frac{FE}{DE}$

$hence\ BE.DE = FE.AE$ (2)

4.3 ∠B1 = 90-x (sum of angles in triangle ABE)

$\therefore D1 = 90 - x$ $(\angle sum\ \Delta)$

$\therefore\ \angle B1 = \angle D1$ (4)