FINANCE

Checklist

Make sure you:

- Understand – nominal & effective interest rates
- Understand the terminology around Depreciation – ‘**Linear/straight line** depreciation’ (simple interest) VS ‘depreciation using the **reducing balance**’ (compound interest)
- understand what an Annuity is, and be able to apply the correct formula
- understanding the *Sinking fund* concept
- can calculate the balance outstanding of an annuity at a point in time

The nominal rate (***nom***) is the rate which is quoted with a compounding interval per annum (***m***).E.g. 10% per annum compounded monthly. **This is a nominal rate.**

The effective rate (*i* **eff**) is the equivalent rate if the interest were to be calculated annually. This is what the nominal rate equates to ***effectively*** over 1 year.

When a rate is quoted without any linked compounding, this rate is the ***effective*** rate.

E.g. An amount of money is invested... at a rate of 10% per annum. **This is an effective rate.**

The challenge that we are sometimes presented with is when a quoted rate is effective, but our calculation requires us to compound, for example, monthly. Then we need to convert from an annual, effective, rate to the equivalent nominal rate, and then substitute into the correct formula, adjusting ***i*** and ***n*** accordingly.

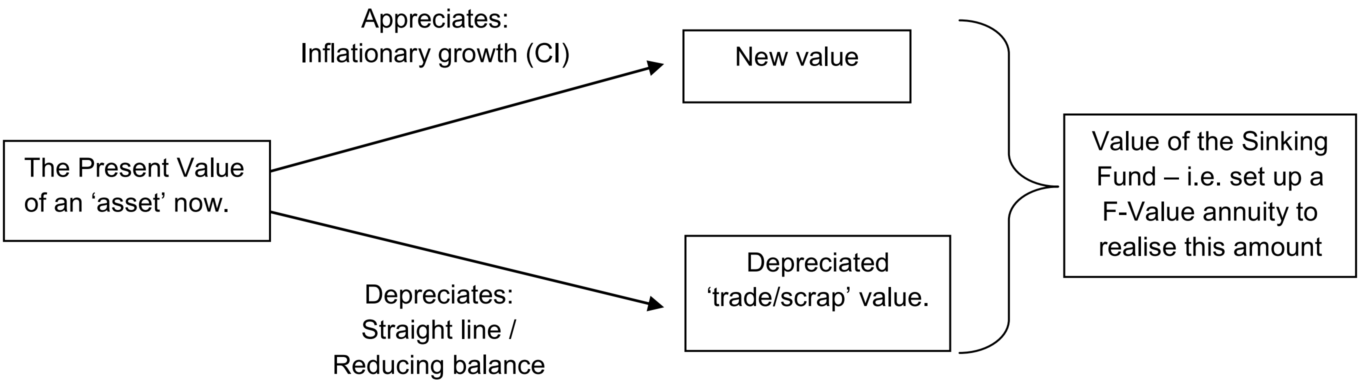
You need to be able to convert the one to the other using this formula: $\left(1+\frac{i_{nom}}{m}\right)^m = 1+\mathrm{i}_{eff}$

An annuity is a set of fixed payments, paid at the same interval using the same interest rate, which usually form an investment (future value) or repay a loan (present value).

If the regular payment is *x* then the formulae for present value and future value are:

$$P_v = x\left[\frac{1-(1+i)^{-n}}{i}\right] \qquad F_v = x\left[\frac{(1+i)^n - 1}{i}\right]$$

Sinking Funds - The basics



Exam Questions

Question 1

Time line question

Leigh-Anne is saving for university and decides to put her money into a fixed deposit paying 10% p.a. compounded annually for the first 5 years. For the reminder of the investment period the interest rate changes to 12% p.a. compounded quarterly. Leigh-Anne starts her savings with R4 000. After 3 years she withdraws R2 000. A final deposit of R8 000 is made 8 years after the initial deposit. How much money has Leigh-Anne accumulated in the fixed deposit at the end of 10 years?

Question 2

Changing in rates – nominal vs effective rates

A bank offers interest on investments at a rate of 15% per annum.

a) Calculate the equivalent nominal interest rate, if calculations are to be performed monthly.

b) Suppose Emma follows the following savings plan (in the bank with an interest rate of 15% per annum.)

1 January 2014:	Deposit	R 2 500
1 April 2014:	Deposit	R 3 000
1 August 2014:	Withdrawal	R 1 800

Determine how much Emma will have in her account at the end of the year.

Question 3

Basic Pv & Fv – instalment with PS & Bal Out

Nosizwe's parents are considering taking a bank loan of R40 000 to cover the costs of her first year at university. They are offered an interest rate of 14,75% p.a. compounded monthly and wish to pay back the loan in 12 equal monthly instalments starting one month after receiving the loan.

a) Calculate what these monthly payments will be.

b) Determine the balance outstanding on the loan, immediately after the 8^{th} payment has been made.

c) If Nosizwe gets a distinction for maths in her final matric examination the University will only require her to pay her first year tuition fees (R40 000) at the end of the first year, interest free! If Nosizwe's parents do not take out the bank loan, but still continue to make the monthly instalments (calculated in part a) into a bank account offering an interest rate of 14,75% p.a. compounded monthly, determine how much extra they will have saved.

Question 4

SF

A new compressor is purchased for a panel-beating shop for an amount of R185 000. If the machine depreciates at a rate of 20% p.a., on the straight line basis, determine:

a) the scrap value of the machinery after 3 years.

b) the cost of a new machine, if the price of new machines increases by 7,2% p.a.

c) the value required in the sinking fund.

d) A sinking fund is set up to realize the amount required to purchase the compressor. If the quoted bank interest rate is 9% p.a. compounded monthly, and payments start immediately, and are then made at the end of each month, determine the value of the monthly payments (over the 3 year period.)

Question 5

Solving for n – Problem Solving

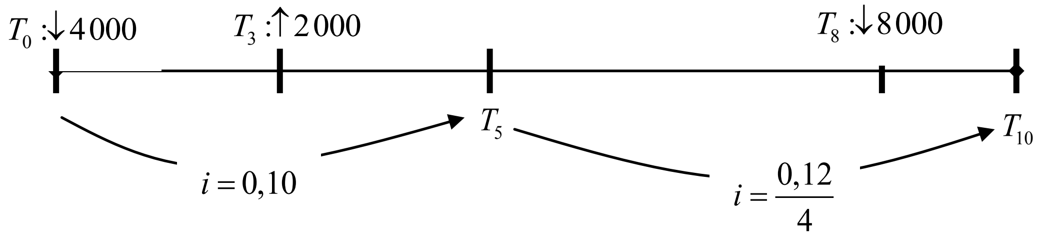
a) How much can you live off, as a monthly allowance, if R100 000 is invested in an account which gains interest at a rate of 8% p.a. compounded monthly and the money must provide income for 2 years only?

b) If you need a monthly income of R3 500 instead, determine how many full withdrawals of R3 500 that can be made.

SOLUTIONS TO FINANCE

Question 1

a) Use a timeline to represent the given information – especially changing interest rates as well as any additional deposits/withdrawals.



$$F = P\left(1+\frac{i}{m}\right)^{m\times n}$$

$$F = 4000(1+0,10)^5\left(1+\frac{0,12}{4}\right)^{5\times4} - 2000(1+0,10)^2\left(1+\frac{0,12}{4}\right)^{5\times4} + 8000\left(1+\frac{0,12}{4}\right)^{2\times4}$$

$$F = R17398,41$$

Question 2

a) start by converting the effective rate to a nominal monthly rate.

$$\left(1+\frac{i_{nom}}{m}\right)^m = 1+i_{eff}$$

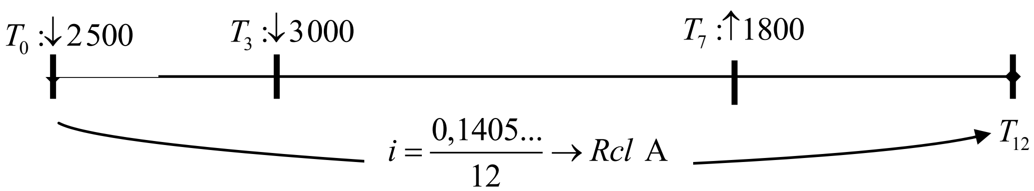
$$\left(1+\frac{i_{nom}}{12}\right)^{12} = 1+0,15$$

$$\therefore \quad \frac{i}{12} = \sqrt[12]{1,15}-1$$

$$\therefore \quad \frac{i}{12} = 0,01171\ldots \quad \text{sto} \rightarrow \text{A}$$

$$\therefore \quad i = 14,057\ldots\% \text{ pa compounded monthly}$$

b)



$$F = 2500(1+\text{A})^{12} + 3000(1+\text{A})^9 - 1800(1+\text{A})^5$$

$$F = \text{R}\,4298,60$$

Question 3

a) $P_v = x\left[\frac{1-(1+i)^{-n}}{i}\right] \therefore 40000 = x\left[\frac{1-\left(1+\frac{0,1475}{12}\right)^{-12}}{\frac{0,1475}{12}}\right]$

$\therefore 40000 = x[11,0938...] \therefore x = \text{R}3605,62$

b) $\text{BO} = 40000\left(1+\frac{0,1475}{12}\right)^{8} - 3605,62\left[\frac{\left(1+\frac{0,1475}{12}\right)^{8}-1}{\frac{0,1475}{12}}\right]$ = R 13989,90

c) $F_v = x\left[\frac{(1+i)^n - 1}{i}\right] \therefore 3605,62\left[\frac{\left(1+\frac{0,1475}{12}\right)^{12}-1}{\frac{0,1475}{12}}\right] = 46315,73$

Total savings = R 46315,73 – R 40 000 = R 6315,73

Question 4

a) $A = P(1-in) = 185000(1-0,2\times 3) = 74000$

b) $F_v = 185000(1+0,072)^3 = 227906,17$ (2 d.p.)

c) $SF_{value} = 227906,17 - 74000 = 153906,17$

d) Note: there are n + 1 monthly payments i.e. n = 37

$$F_v = x\left[\frac{(1+i)^n - 1}{i}\right] \therefore x\left[\frac{\left(1+\frac{0,09}{12}\right)^{37}-1}{\frac{0,09}{12}}\right] = 153906,17$$

$$x(42,46...) = 153906,17$$

$$\therefore x = 3624,62$$

Question 5

a) $P_v = x\left[\dfrac{1-(1+i)^{-n}}{i}\right]$ $\therefore 100000 = x\left[\dfrac{1-\left(1+\dfrac{0,08}{12}\right)^{-24}}{\dfrac{0,08}{12}}\right]$ $\therefore\ x = \dfrac{100000}{22,11...} = 4522,73$

b) $100000 = 3500\left[\dfrac{1-\left(1+\dfrac{0,08}{12}\right)^{-n}}{\dfrac{0,08}{12}}\right]$ $\therefore\ \dfrac{100000}{3500}\times\dfrac{0,08}{12} = 1-\left(1+\dfrac{0,08}{12}\right)^{-n}$

$$\therefore\ \frac{17}{21} = \left(\frac{151}{150}\right)^{-n}$$

$$\therefore\ \log_{\frac{151}{150}}\frac{17}{21} = -n$$

$$\therefore\quad n = 31,80...$$

i.e. 31 full payments of R3 500.