NUMBER PATTERNS

Checklist

Make sure you:

- Know how to **distinguish** between an arithmetic and geometric sequence or series.
- Know how to **derive** and **apply** the formulae for the sum of arithmetic and geometric series.
- Know how to **identify** and **apply** the formulae for the terms of arithmetic and geometric sequences.
- Know how to find the **next term** in an arithmetic and geometric sequence.
- Can find the terms and the sum of a series which given in **sigma notation**.

Exam Questions

Question 1

(Adapted from DBE Feb 2014, Paper 1, Question 2)

A geometric sequence has T3=20 and T4=40

Determine:

1.1. The common ratio (1)

1.2. A formula for T_n (3)

Question 2

(Adapted from DBE Nov 2013, Paper 1, Question 2).

Given the geometric sequence

7; x; 63;…

Determine the possible values of x (3)

Question 3

(Adapted from Feb 2014, Paper 1, Question 3)

Given the arithmetic sequence:

w-3; 2w-4; 23-w

3.1 Determine the value of w (2)

3.2. Write down the common difference of this sequence. (1)

Question 4

(Adapted from Feb 2014, Paper 1, Question 3)

The arithmetic sequence 4;10;16;… is the sequence of first differences of a quadratic sequence with a first term equal to 3.

Determine the 50^{th} term of the quadratic sequence. (5)

Question 5

(Adapted from Feb 2014, Paper 1, Question 2)

The following sequence has the property that the sequence of numerators is arithmetic and the sequence of denominators is geometric:

2/1;-1/5;-4/25;…

5.1. Write down the FOURTH term of the sequence. (1)

5.2. Determine a formula for the nth term. (3)

5.3 Determine the 500^{th} term of the sequence. (2)

5.4 Which will be the first term of the sequence to have a NUMERATOR which is less than -59? (3)

Question 6

(Adapted from Nov 2013, Paper 1, Question 2)

Given:

0; - 1/2;0; 1/2;0; 3/2;0; 5/2;0; 7/2;0;…

Assume that this number pattern continues consistently.

6.1. Write down the value of the 191^{st} term of this sequence. (1)

6.2. Determine the sum of the first 500 terms of this sequence. (4)

Question 7

(Adapted from Nov 2013, Paper 1, Question 2)

Given: $\sum_{k=2}^{20}(4x-1)^k$

7.1. Calculate the first term of the series $\sum_{k=2}^{20}(4x-1)^k$ if $x=1$. (2)

7.2. For which values of x will $\sum_{k=2}^{\infty}(4x-1)^k$ exist? (3)

Question 8

(Adapted from Feb 2014, Paper 1, Question 4)

In a geometric series, the sum of the first n terms is given by $S_n = p\left(1-\left(\frac{1}{2}\right)^n\right)$ and the sum to infinity of the series is 10.

8.1. Calculate the value of p. (4)

8.2. Calculate the second term of the series. (4)

SOLUTIONS TO NUMBER PATTERNS

Question 1

1.1 $r = T_4 \div T_3 = 40 \div 20 = 2$

1.2 $T_n = ar^{n-1}$

$\therefore 20 = a(2)^{3-1}$

$\therefore a = 5$

$\therefore T_n = 5.2^{n-1}$

Question 2

$$\frac{T_2}{T_1} = \frac{T_2}{T_1}$$

$$\therefore \frac{x}{7} = \frac{63}{x}$$

$$\therefore x^2 - 441 = 0$$

$$\therefore x = -21 \text{ or } x = 21$$

Question 3

3.1 $T_2 - T_1 = T_3 - T_2$

$(2w-4) - (w-3) = (23-w) - (2w-4)$

$w - 1 = 27 - 3w$

$\therefore w = 7$

3.2 $w-3;\ 2w-4;\ 23-w$

$4;\ 10;\ 16$

$d = T_2 - T_1 = 10 - 4 = 6$

Question 4

a + b + c: 3 ; 7 ; 17 ; 33

3a + b: 4; 10; 16

2a: 6 6

$T_n = an^2 + bn + c$

$2a = 6$ $3a+b = 4$ $a + b + c = 3$

$\therefore a = 3$ $\therefore b = -5$ $\therefore c = 5$

$T_n = 3n^2 - 5n + 5$

$T_{50} = 3(50)^2 + -5(50) + 5 = 7$

Question 5

5.1 $T_4 = \frac{-7}{125}$

5.2 $T_n = \frac{a+(n-1)d}{a.r^{n-1}}$

$\therefore T_n = \frac{2+(n-1)(-3)}{1.5^{n-1}}$

$\therefore T_n = \frac{5-3n}{5^{n-1}}$

5.3 $T_{500} = \frac{5-3(500)}{5^{500-1}}$

$= \frac{-1\,495}{5^{499}}$

5.4 $5 - 3n < -59$

$-3n < -64$

$n > 21{,}333\ldots$

$n = 22$

Question 6

6.1 0

6.2 $S_{500} = \frac{250}{2}\left[2\left(-\frac{1}{2}\right) + (250-1)(1)\right]$

$= 31\,000$

Question 7

7.1 $T_1 = (4(1)-1)^2 = 9$

7.2. $-1 < r < 1$

$-1 < 4x - 1 < 1$

$0 < 4x < 2$

$0 < x < \frac{1}{2}$

Question 8

8.1 $S_n = p\left(1 - \left(\frac{1}{2}\right)^n\right)$

$a = p\left[1 - \left(\frac{1}{2}\right)^1\right]$

$a = \frac{p}{2}$

$$r = \frac{1}{2}$$

$$\therefore 10 = \frac{\frac{p}{2}}{1 - \frac{1}{2}}$$

$$5 = \frac{p}{2}$$

$$p = 10$$

8.2. $r = \frac{1}{2}$

$$\frac{a}{1 - \frac{1}{2}} = 10$$

$$a = 5$$

$$T_2 = ar = \frac{5}{2}$$