LIVE: FINAL EXAM PREPARATION PAPER 1 30 OCTOBER 2014

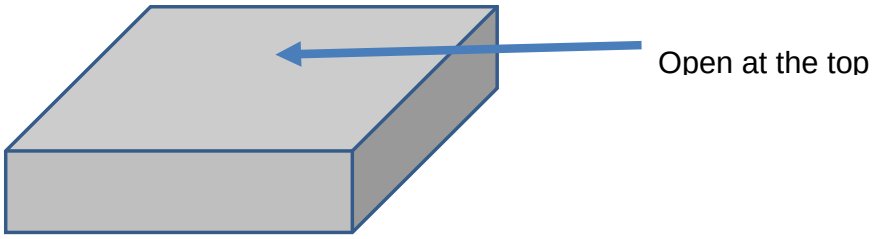
Lesson Description

In this lesson we:

- Work through questions from various Paper 1 papers.

Challenge Question

Philip is designing large fish troughs in the shape of rectangular prisms with an open top as shown in the diagram below. The table shows some standard sized troughs and their surface areas.



Fish Trough	Surface Area							
1	$2[1^2 + 3(1)]$	+	$2[1^2 + 5(1)]$	+	$(1 + 3)(1 + 5)$	=	44	m^2
2	$2[2^2 + 3(2)]$	+	$2[2^2 + 5(2)]$	+	$(2 + 3)(2 + 5)$	=	83	m^2
3	$2[3^2 + 3(3)]$	+	$2[3^2 + 5(3)]$	+	$(3 + 3)(3 + 5)$	=	132	m^2
4	$2[4^2 + 3(4)]$	+	$2[4^2 + 5(4)]$	+	$(4 + 3)(4 + 5)$	=	191	m^2
5	$2[5^2 + 3(5)]$	+	$2[5^2 + 5(5)]$	+	$(5 + 3)(5 + 5)$	=	260	m^2
m	A	+	B	+	C	=	D	m^2

(a) Find the value of A + B + C in terms of *m.*

(Place your answer into simplest form.) (5)

(b) Philip's fish troughs are all built using the surface area formula of A + B + C. What is the surface area of a trough so that the trough can hold 1 512 m^3 of water if filled right to the top? (5)

Exam Questions

Question 1

(a) Give the next two terms in the sequence, assuming that it remains consistent:

$$\frac{1}{3},\ \frac{3}{5},\ \frac{5}{7},\ \ldots$$ (1)

(b) Solve for x:

(1) $x^2 - 3x = 2(x - 1)$ (correct to one decimal digit) (5)

(2) $8^{x-5} = 32^{10-2x}$ (4)

(3) $\sqrt[3]{3x - 23} = -2$ (2)

(c) Check whether $x = -1$ is a root of the equation:

$3x^7 - 2x = x^3 + 1$ (3)

(d) Simplify: $\dfrac{p+q}{p-q} - \dfrac{p-q}{p+q}$ (3)

(e) Determine the 60th term of the geometric sequence, leaving your answer in exponential form.

$$10,\ -5,\ \frac{5}{2},\ -\frac{5}{4},\ \ldots$$ (2)

(f) Find n so that $\sum_{k=1}^{n}(3k - 10) = 8018$ (7)

(g) Given: $f(x) = x^3 - x^2 - 22x + 40$

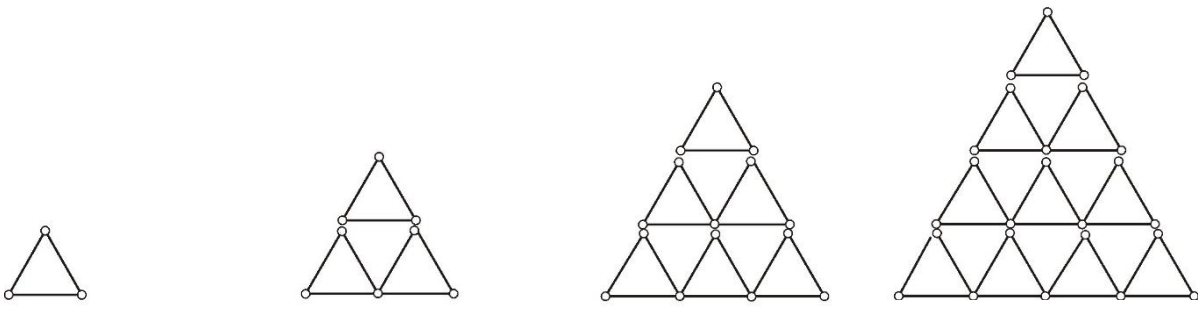
(1) Use the Factor Theorem to fully factorise $f(x)$. (5)

(2) Determine $f'(1)$. (3)

Question 2

(a) Refer to the figure showing stacking of congruent triangles.

Fig. 1 Fig.2 Fig.3 Fig.4



(1) Complete the table:

Figure	1	2	3	4	5
No. of sides	3		18	30	

(2)

(2) Determine a formula for the number of sides in the n^{th} figure. (7)

(3) Hence determine which figure has 900 sides of triangles. (4)

Qestion 3

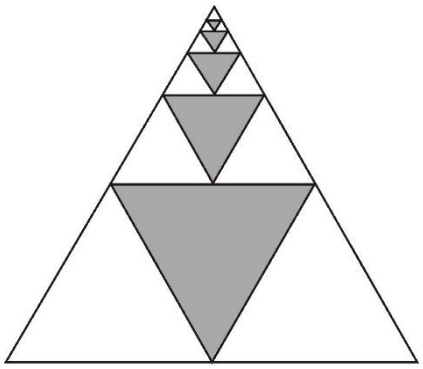
Refer to the figure.

The largest triangle has an area of one square unit.

The biggest grey triangle has area $\frac{1}{4}$ sq. units and each subsequent triangle's area is $\frac{1}{4}$ the size of the triangle bigger than it.

These triangles continue indefinitely.

Determine the area of the unshaded part of the triangle



Question 4

(a) Determine $\frac{dy}{dx}$ for $y = \frac{x^3 + 6x^2 - 5x + 4}{x}$ (4)

(b) Given: $f(x) = x^3 - 4x$, find $f'\left(2\sqrt{2}\right)$ (3)

Question 5

(a) If $a + 2b = 5$ and $c = 3$, what is the value of $a + 2(b + c)$? (2)

(b) Given: $f(x) = 9$ and $g(x) = x^2$

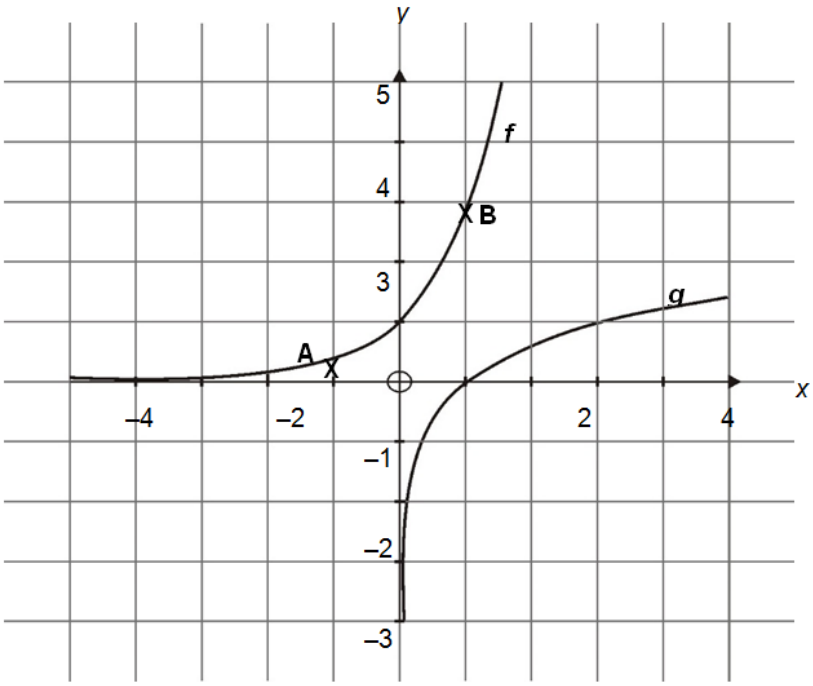
(1) State the range of each function. (2)

(2) Determine the value(s) of x for which $f(x) > g(x)$. (2)

Question 6

Refer to the figure below showing the graphs of

$f(x) = 3^x$ and $g(x) = f^{-1}(x)$.



(a) Calculate the average gradient of the curve of f between the points A (where $x = -1$) and B (where $x = 1$). (2)

(b) Give the values of x for which $f'(x) > 0$. (1)

(c) Give the equation of g in the form $g(x) = \ldots$ (2)

Question 7

Given: $f(x) = \dfrac{3}{x-2} - 1$ and $g(x) = 3x - 7$

(a) Draw the graphs of f and g.

All the intercepts with the axes and asymptotes should be clearly shown. (7)

(b) Use your graphs to determine the value(s) of x for which:

(1) $f(x) = g(x)$ (1)

(2) $f(x) > g(x)$ (2)

Question 8

(a) Clare needed R500 urgently. A 'loan shark' agreed to give it to her for one month but she would have to return R600 to him.

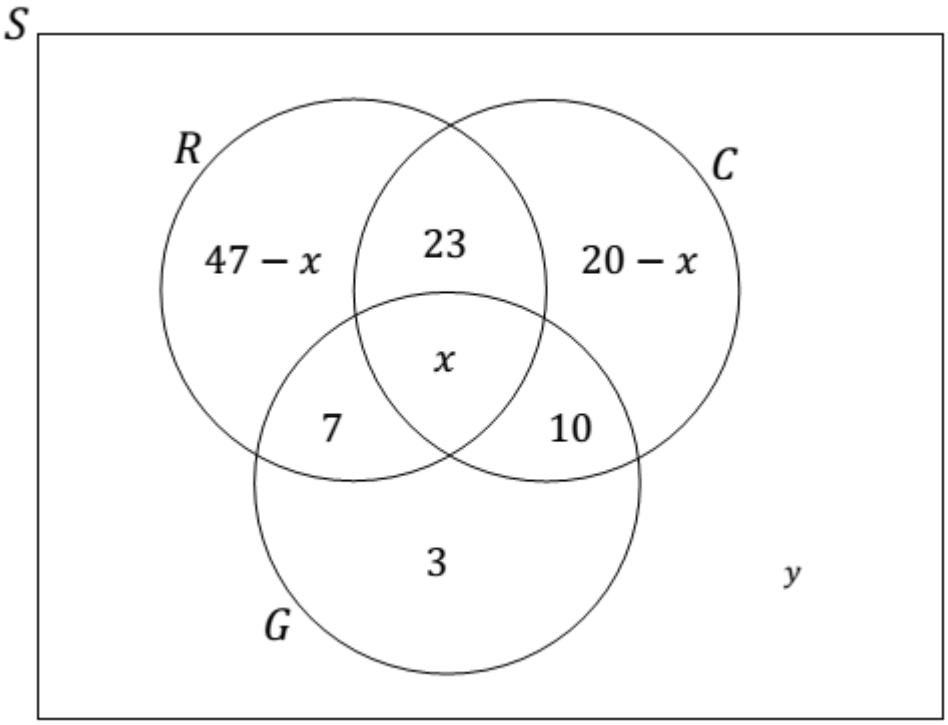
(1) Determine the interest rate that he is charging for this one month loan. (2)

(2) Show that if this monthly rate is compounded for 12 months then it is equivalent to an effective annual percentage rate of nearly 800%. (4)

(b) The owner of a small business decides that in one month's time he must start depositing R2 000 per month into a sinking fund earning 10,25% p.a. compounded monthly in order to be able to replace his power generator. It is expected to cost R34 000. Calculate how many months it will take before he has sufficient funds. (8)

Question 9

There are 115 people in a group. The Venn-diagram below shows the number of people who enjoy listening to radio (R), enjoy gardening (G) and/or enjoy cooking (C). There are people who enjoy all three activities. There arepeople who do not enjoy any of the activities.



9.1 If there are 28 people who enjoy gardening, calculate the value of x (1)

9.2 Hence determine the value of y (1)

Question 10

The 9 letters in the word CELLPHONE are each written on a card and rearranged.

10.1 How many different arrangements can be made if the repeated letters (E and L) are considered as different? (1)

10.2 Determine the probability that the two E's will be placed next to each other if the repeated letters are considered as different. (3)

10.3 How many different arrangements that start with the letter P can be made if the repeated letters are considered as the same? (2)

Answers

Exam Questions

Question 1

(a) $\frac{7}{9}; \frac{9}{11}$ ✓A

(b) (1) $x^2 - 3x = 2(x - 1)$

$= 2x - 2$ ✓M

$x^2 - 5x + 2 = 0$ ✓A

$x = \frac{5 \pm \sqrt{25 - 8}}{2}$ ✓M

$= \frac{5 \pm \sqrt{17}}{2}$ ✓A

$= 4{,}6 \text{ or } 0{,}4$ ✓CA

(2) $8^{x-5} = 32^{10-2x}$

$2^{3x-15} = 2^{50-10x}$ ✓M ✓A

$3x - 15 = 50 - 10x$ ✓M

$13x = 65$

$x = 5$ ✓A

(3) $\sqrt[3]{3x - 23} = -2$

$3x - 23 = -8$ ✓M

$3x = 15$

$x = 5$ ✓A

(c) LHS $= 3(-1)^7 - 2(-1)$ RHS $= (-1)^3 + 1$

$= -3 + 2$ $= -1 + 1$

$= -1$ $= 0$

✓A ✓A

$\therefore$ $x = -1$ is NOT a root. ✓CA

(d) $\dfrac{p+q}{p-q} - \dfrac{p-q}{p+q}$

$= \dfrac{(p+q)^2 - (p-q)^2}{(p-q)(p+q)}$ ✓A

$= \dfrac{p^2 + 2pq + q^2 - (p^2 - 2pq + q^2)}{(p-q)(p+q)}$ ✓M

$= \dfrac{4pq}{(p-q)(p+q)}$ ✓A

(e) 10, –5, $\dfrac{5}{2}$, $-\dfrac{5}{4}$, …

G.P. $a = 10$, $r = -\dfrac{1}{2}$ ✓A

$T_{60} = ar^{59}$

$= 10\left(-\dfrac{1}{2}\right)^{59}$ ✓A

(f) $\sum_{k=1}^{n}(3k - 10)$

$= -7 - 4 - 1 + 2 + \ldots + 3n - 10$ ✓M

A.P. $a = -7$, $d = 3$, $S_n = 8018$ ✓A

$\dfrac{n}{2}\{2(-7) + (n-1)3\} = 8018$ ✓M

$n\{-14 + 3n - 3\} = 16036$

$3n^2 - 17n - 16036 = 0$ ✓A

$n = \dfrac{17 \pm \sqrt{(-17)^2 + 4(3)16036}}{6}$ ✓M

$= 76$ or $\dfrac{-211}{3}$ ✓A

i.e. $n = 76$ ✓A

(g) $f(x) = x^3 - x^2 - 22x + 40$

(1) $f(2) = 2^3 - 2^2 - 22 \times 2 + 40$ ✓M

$= 8 - 4 - 44 + 40$

$= 0$

$\therefore x - 2$ is a factor ✓A

$f(x) = (x - 2)(x^2 + x - 20)$ ✓M ✓A

$= (x - 2)(x + 5)(x - 4)$ ✓A

(2) $f(x) = x^3 - x^2 - 22x + 40$

$f'(x) = 3x^2 - 2x - 22$ ✓A

$f'(1) = 3 - 2 - 22$ ✓M

$= -21$ ✓A

Question 2

(a) (1)

Figure	1	2	3	4	5
No. of sides	3	**9**	18	30	**45**

✓A ✓A

(2) 1st diff.: 6 9 12 15

2nd diff.: 3 3 3 ✓M ✓A

$\therefore$ Formula of form: $T_n = an^2 + bn + c$

$2a = 3$ $T_0 = 0$

$a = \frac{3}{2}$ $c = 0$ ✓A ✓A

$\therefore T_n = \frac{3n^2}{2} + bn$

$T_1 = \frac{3}{2} + b = 3$ ✓M

Question 3

Shaded area forms G.S. $a = r = \frac{1}{4}$ ✓A

$S_\infty = \frac{\frac{1}{4}}{1 - \frac{1}{4}}$ ✓A

$= \frac{1}{4} \div \frac{3}{4}$ ✓M

$= \frac{1}{3}$ ✓A

$\therefore$ Unshaded part is $\frac{2}{3}$ sq. units ✓CA

Question 4

a) $y = \frac{x^3 + 6x^2 - 5x + 4}{x}$

$= x^2 + 6x - 5 + 4x^{-1}$ ✓M ✓A

$\frac{dy}{dx} = 2x + 6 - 4x^{-2}$ ✓M

$= 2x + 6 - \frac{4}{x^2}$ ✓A

(b) $f(x) = x^3 - 4x$

$f'(x) = 3x^2 - 4$ ✓A

$f'\left(2\sqrt{2}\right) = 3\left(2\sqrt{2}\right)^2 - 4$ ✓M

$= 3 \times 8 - 4$

$= 20$ ✓A

Question 5

(a) (1) $a + 2b + 2c$

$= 5 + 2 \times 3$ ✓M

$= 11$ ✓A

(b) (1) f: Range: $y = 9$ ✓A

g: Range: $y \geq 0$ ✓A

(2) $f(x) > g(x)$

$9 > x^2$ ✓A

$-3 < x < 3$ ✓A

Question 6

(a) (1) Ave. Grad. $= \frac{f(1) - f(-1)}{1 - (-1)}$

$= \frac{3 - \frac{1}{3}}{2}$ ✓M

$= \frac{4}{3}$ ✓A

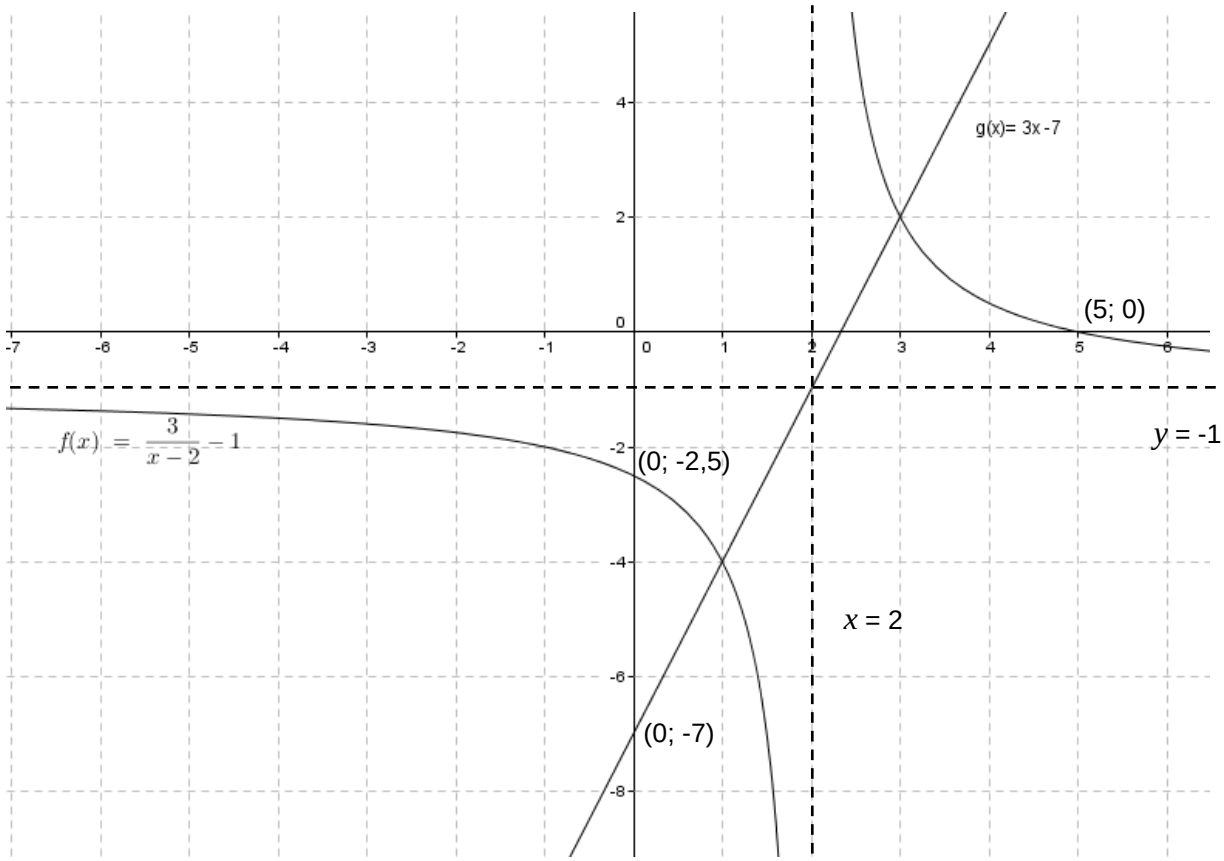
(2) $x \in R$ ✓A

(3) $g(x) = \log_3 x$ ✓M ✓A

(4) $A'(-2; \frac{1}{3})$ $B'(0; 3)$ ✓A ✓A

Question 7

a.)



b.) (1) (1; -4) and (3; 2)

(2) x <1 and 2 < x < 3

Question 8

(a) (1) Interest = R100

$\frac{100}{500} \times 100$ ✓M

= 20% per month ✓A

(2) $1 + i = (1, 2)^{12}$ ✓M ✓A

$= 8,9161 \ldots$ ✓A

$i = 7,9161 \ldots$

$\approx$ 791,6% p.a. (nearly 800% p.a.!) ✓CA

(b) $\left(1+\frac{0{,}1025}{12}\right) = 1{,}00854667 = A$ ✓A

$\frac{x\left[(1+i)^n-1\right]}{i} = 34\,000$ ✓M

$\frac{2000\left[A^n-1\right]}{A} = 34\,000$ ✓A

$(A)^n - 1 = 0{,}1452083$ ✓M

$(A)n = 1{,}145\ldots$ ✓A

$n = \log_A 1{,}145\ldots$ ✓M

$= 15{,}94124696\ldots$ ✓A

i.e. 16 months ✓CA

Question 9

9.1 $x = 8$

9.2 $y = 13$

Question 10

10.1 9! = 362 880

10.2 n(E's next to each other) = 8 x 7! X 2 = 80 640

P ((E's next to each other) = 80 640 ÷ 362 880 = 2÷9

10.3 n(start with the letter P, repeated letters the same) = 8! ÷ 2!.2! = 10 080