LIVE: PAPER 2 QUESTIONS

Lesson Description

In this lesson we:

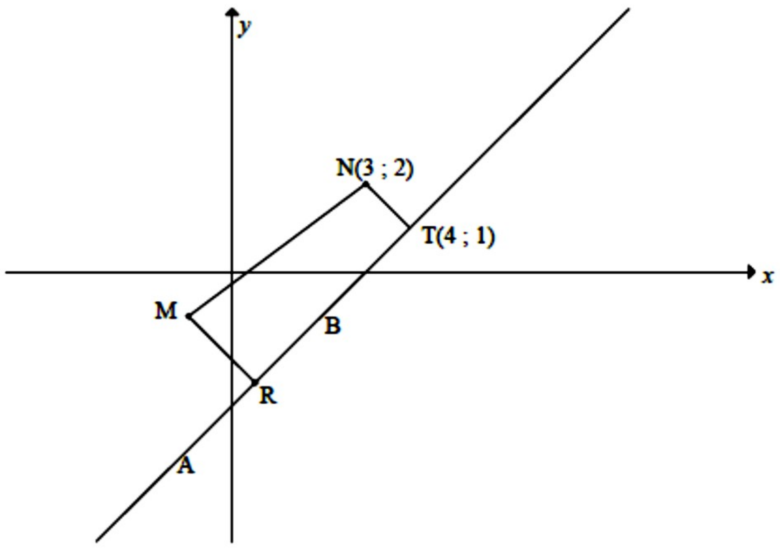
- Work through selected examination questions adapted from 2014 Exemplar Paper covering:
    - o Analytical Geometry
    - o Euclidean Geometry
    - o Trigonometry
    - o Statistics

Improve your Skills

Question 1

(Adapted from DBE 2014 Exemplar P2, Question 4)

In the diagram below, the equation of the circle having centre M is $(x + 1)^2 + (y + 1)^2 = 9$. R is a point on chord AB such that MR bisects AB. ABT is a tangent to the circle having centre N(3 ; 2) at point T(4 ; 1).



1.1 Write down the coordinates of M. (1)

1.2 Determine the equation of AT in the form y = mx + c. (5)

1.3 If it is further given that MR = $\frac{\sqrt{10}}{2}$ units, calculate the length of AB. Leave your answer in simplest surd form. (4)

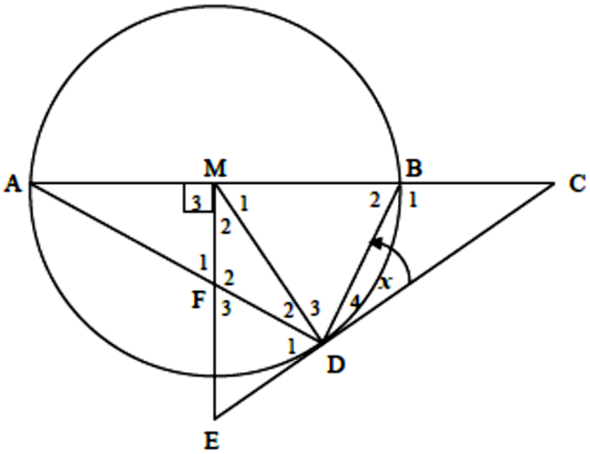
1.4 Calculate the length of MN. (2)

1.5 Another circle having centre N touches the circle having centre M at point K. Determine the equation of the new circle. Write your answer in the form $x^2 + y^2 + Cx + Dy + E = 0$. (3)

Question 2

(Adapted from DBE 2014 Exemplar P2, Question 9)

In the diagram, M is the centre of the circle and diameter AB is produced to C. ME is drawn perpendicular to AC such that CDE is a tangent to the circle at D. ME and chord AD intersect at F. MB = 2BC.

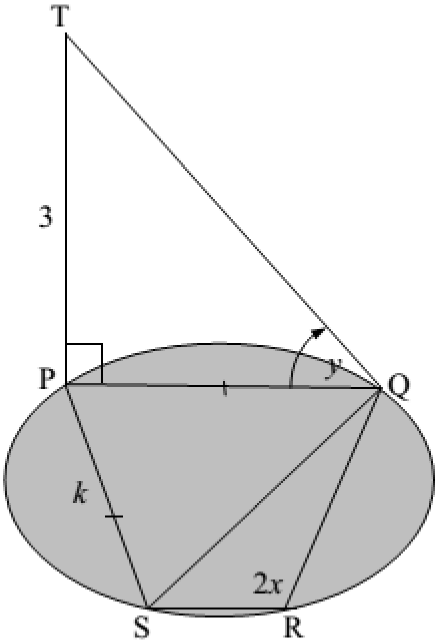


2.1	If angle $D_4 = x$, write down, with reasons, TWO other angles each equal to x.	(3)
2.2	Prove that CM is a tangent at M to the circle passing through M, E and D.	(4)
3.3	Prove that FMBD is a cyclic quadrilateral.	(3)
2.4	Prove that $DC^2 = 5BC^2$.	(3)
2.5	Prove that $\Delta DBC \mid\mid\mid \Delta DFM$.	(4)
2.6	Hence, determine the value of DM ÷ FM	(2)

Question 3

(Adapted from DBE 2014 Exemplar P2, Question 7.2)

The framework for a construction consists of a cyclic quadrilateral PQRS in the horizontal plane and a vertical post TP as shown in the figure. From Q the angle of elevation of T is y°. PQ = PS = k units, TP = 3 units and angle SRQ = 2x°



3.1 Show, giving reasons, that $x = P\hat{S}Q$. (2)

3.2 Prove that SQ = 2k cos x. (4)

3.3 Hence, prove that $SQ = \dfrac{6\cos x}{\tan y}$ (2)

Question 4

(Adapted from DBE 2014 Exemplar P2, Question 5.3)

Determine the general solution of $\cos 2\theta + 4\sin^2\theta - 5\sin\theta - 4 = 0$. (7)

Question 5

(Adapted from DBE 2014 Exemplar P2, Question 2)

The table below shows the amount of time (in hours) that learners aged between 14 and 18 spent watching television during 3 weeks of the holiday.

Time (hours)	Cumulative frequency
$0 \leq t < 20$	25
$20 \leq t < 40$	69
$40 \leq t < 60$	129
$60 \leq t < 80$	157
$80 \leq t < 100$	166
$100 \leq t < 120$	172

5.1 Draw an ogive (cumulative frequency curve) to represent the above data. (3)

5.2 Write down the modal class of the data. (1)

5.3 Use the ogive (cumulative frequency curve) to estimate the number of learners who watched television more than 80% of the time. (2)

5.4 Estimate the mean time (in hours) that learners spent watching television during 3 weeks of the holiday. (4)

SOLUTIONS TO PAPER 2 QUESTIONS (LIVE)

Question 1

1.1 M(–1 ; –1)

1.2

$$m_{NT} = \frac{2-1}{3-4} = -1$$

$$\therefore\ m_{AT} = 1 \qquad \text{(radius} \perp \text{tangent)}$$

$$y - 1 = 1(x-4)$$

$$y = x - 3$$

1.3

$$\text{MR} \perp \text{AB} \qquad \text{(line from centre to midpt of chord)}$$

$$\text{MB}^2 = \text{MR}^2 + \text{RB}^2 \qquad \text{(Theorem of Pythagoras)}$$

$$9 = \left(\frac{\sqrt{10}}{2}\right)^2 + \text{RB}^2$$

$$\text{RB}^2 = \frac{13}{2}$$

$$\text{RB} = \sqrt{\frac{13}{2}}$$

$$\text{AB} = 2\left(\sqrt{\frac{13}{2}}\right) = \sqrt{26}\ \textit{units}$$

1.4

$$\text{MN}^2 = (-1-3)^2 + (-1-2)^2$$

$$= 16 + 9$$

$$= 25$$

$$\text{MN} = 5 \text{ units}$$

1.5

$$r = 5 - 3 = 2 \text{ units}$$

$$\therefore (x-3)^2 + (y-2)^2 = 4$$

$$\therefore x^2 + y^2 - 6x - 4y + 9 = 0$$

Question 2

2.1 $\hat{D}_4 = \hat{A} = x$ (tan chord theorem)

$\hat{A} = \hat{D}_2 = x$ ($\angle$s opp equal sides)

2.2 $\hat{M}_1 = 2x$ (ext $\angle$ of Δ) or ($\angle$ at centre = 2$\angle$ at circum)

$M\hat{D}E = 90°$ (radius $\perp$ tan)

$\hat{M}_2 = 90° - 2x$

$\therefore\ \hat{E} = 180° - (90° + 90° - 2x)$ (sum of $\angle$s in ΔMDE)

$= 2x$

$\therefore$ CM is a tangent (converse tan chord theorem)

2.3 $\hat{M}_3 = 90°$ (EM $\perp$ AC)

$A\hat{D}B = 90°$ ($\angle$ in semi-circle)

$\therefore$ FMBD a cyclic quad (ext $\angle$ of quad = int opp $\angle$)

OR

$E\hat{M}C = 90°$ (EM $\perp$ AC)

$A\hat{D}B = 90°$ ($\angle$ in semi-circle)

$\therefore$ FMBD a cyclic quad (opp $\angle$s of quad supp)

2.4 $DC^2 = MC^2 - MD^2$ (Theorem of Pythagoras)

$= (3BC)^2 - (2BC)^2$ (MB = MD = radii)

$= 9BC^2 - 4BC^2$

$= 5BC^2$

2.5

In ΔDBC and ΔDFM:

$\hat{D}_4 = \hat{D}_2 = x$ (proven in 9.1)

$\hat{B}_1 = \hat{F}_2$ (ext $\angle$ of cyclic quad)

$\hat{C} = \hat{M}_2$

$\therefore \Delta DBC\ |||\ \Delta DFM$ ($\angle$; $\angle$; $\angle$)

2.6 $\frac{DM}{FM}=\frac{DC}{BC}$ $(\Delta DBC\ |||\ \Delta DFM)$

$=\frac{\sqrt{5}BC}{BC}$

$=\sqrt{5}$

Question 3

3.1 $S\hat{P}Q=180°-2x$ (opp $\angle$s of cyclic quad)

$P\hat{S}Q+P\hat{Q}S=2x$ (sum of $\angle$s in Δ)

$P\hat{S}Q=P\hat{Q}S=x$ ($\angle$s opp equal sides)

3.2
$$\frac{\sin S\hat{P}Q}{SQ}=\frac{\sin P\hat{S}Q}{PQ}$$
$$\frac{\sin(180°-2x)}{SQ}=\frac{\sin x}{k}$$
$$SQ=\frac{k\sin 2x}{\sin x}$$
$$SQ=\frac{k(2\sin x.\cos x)}{\sin x}=2k\cos x$$

3.3
$$\tan y=\frac{3}{k}$$
$$k=\frac{3}{\tan y}$$
$$\therefore SQ=2\cos x\left(\frac{3}{\tan y}\right)$$
$$=\frac{6\cos x}{\tan y}$$

Question 4

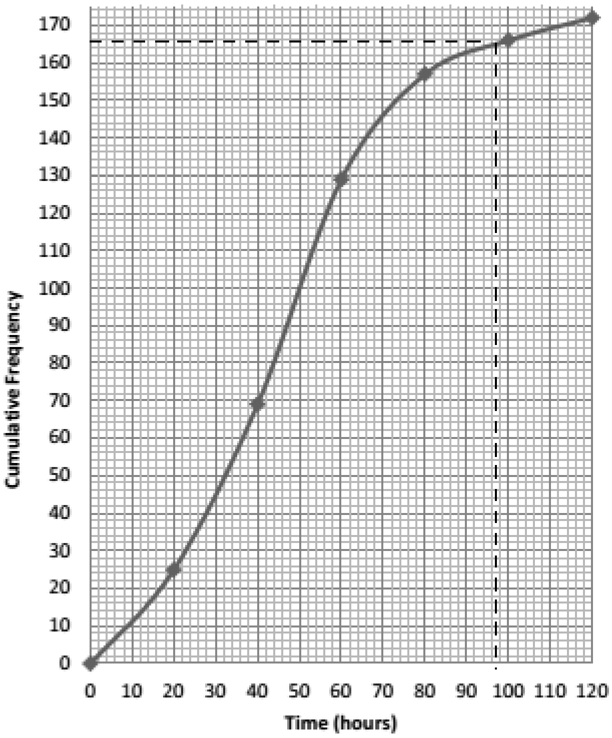
$$1-2\sin^2\theta+4\sin^2\theta-5\sin\theta-4=0$$
$$2\sin^2\theta-5\sin\theta-3=0$$
$$(2\sin\theta+1)(\sin\theta-3)=0$$

$\therefore\ \sin\theta=-\frac{1}{2}$ or $\sin\theta=3$ (no solution)

$\therefore\theta=210°+360°k$ or $\theta=330°+360°k\ \ ;k\in Z$

Question 5

5.1



5.2 $40 \leq t < 60$

5.3 (96 ; 164)

∴ 172 – 164 = 8 learners

5.4 Frequency: 25; 44; 60; 28; 9; 6

$$\text{Mean} = \frac{25 \times 10 + 44 \times 30 + 60 \times 50 + 28 \times 70 + 9 \times 90 + 6 \times 110}{172}$$

= 8000 ÷ 172

= 46,51 hours