TRIGONOMETRY: 2D & 3D PROBLEMS 13 APRIL 2015

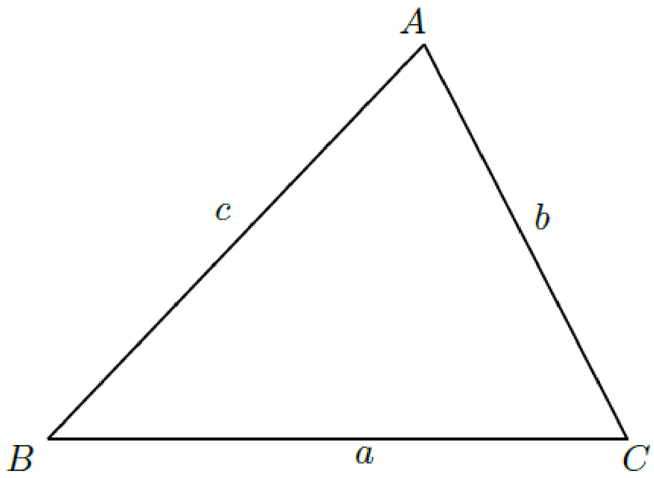
Section A: Summary Notes

Solving two-dimensional problems using the sine, cosine and area rules

- The **sine-rule** is used when the following is known in a triangle which is not a right angled triangle:

  - 2 angles and a side

  - 2 sides and an angle (not included)



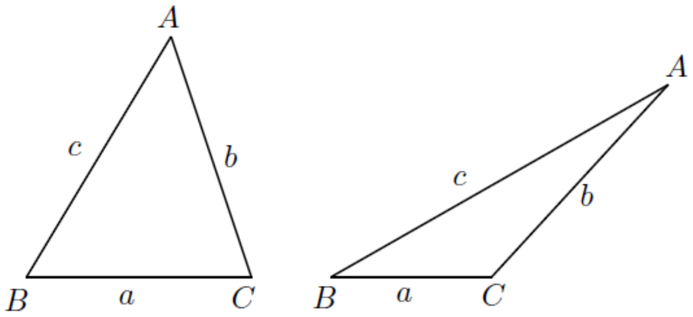
$$\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b} = \frac{\sin \hat{C}}{c}$$

$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

- The **cosine-rule** is used when the following is known in a triangle which is not a right angled triangle:

  - 3 sides

  - 2 sides and an included angle

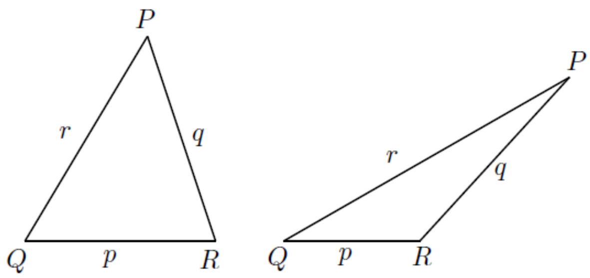


$$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$$

$$b^2 = a^2 + c^2 - 2ac\cos\hat{B}$$

$$c^2 = a^2 + b^2 - 2ab\cos\hat{C}$$

- The **area** of any triangle can be found when at least two sides and an included angle are known



$$\begin{aligned}\text{Area } \triangle PQR &= \frac{1}{2}qr\sin\hat{P}\\ &= \frac{1}{2}pr\sin\hat{Q}\\ &= \frac{1}{2}pq\sin\hat{R}\end{aligned}$$

Section B: Exam practice questions

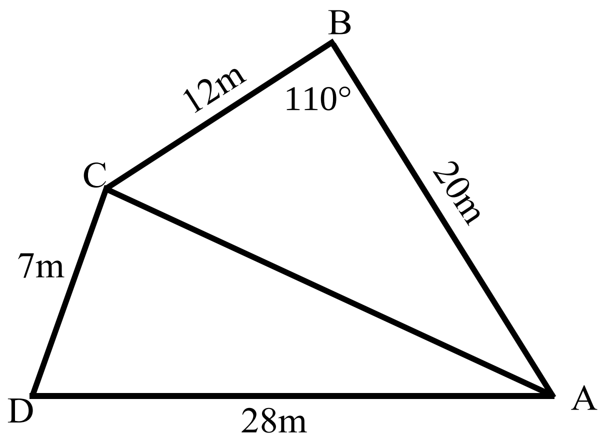
Question 1

A piece of land has the form of a quadrilateral ABCD with $AB = 20m,\ BC = 12m$, and.

$CD = 7m$ and $AD = 28m$

The owner decides to divide the land into two plots by erecting a fence from A to C

$\hat{B} = 110°$



1.1 Calculate the length of the fence AC correct to one decimal place. (2)

1.2 Calculate the size of $B\hat{A}C$ correct to the nearest degree. (2)

1.3 Calculate the size of $\hat{D}$, correct to the nearest degree. (3)

1.4 Calculate the area of the entire piece of land ABCD, correct to one decimal place. (3)

Question 2

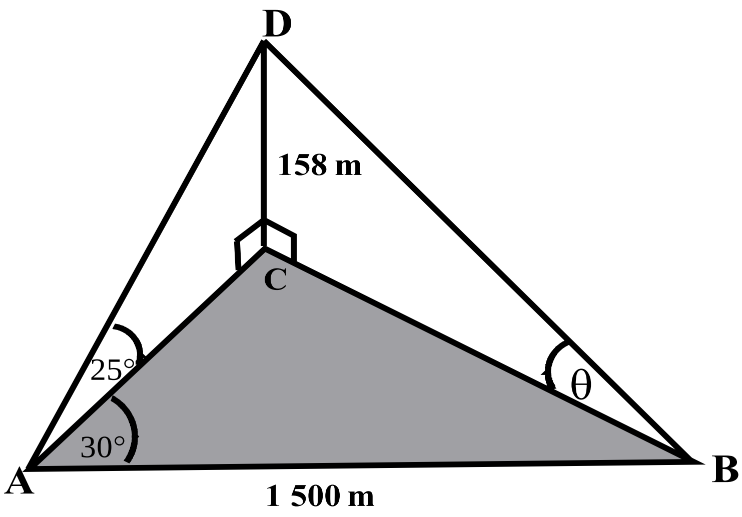
In the diagram below, AB is a straight line 1 500 m long. DC is a vertical tower 158 metres high with C, A and B points in the same horizontal plane. The angles of elevation of D from A and B are $25°$ and θ. Also $C\hat{A}B = 30°$.

2.1 Determine the length of AC. (3)

2.2 Find the value of θ. (5)

2.3 Calculate the area of ΔABC. (2)

2.4 Calculate the size of $A\hat{D}B$ (6)



Question 3

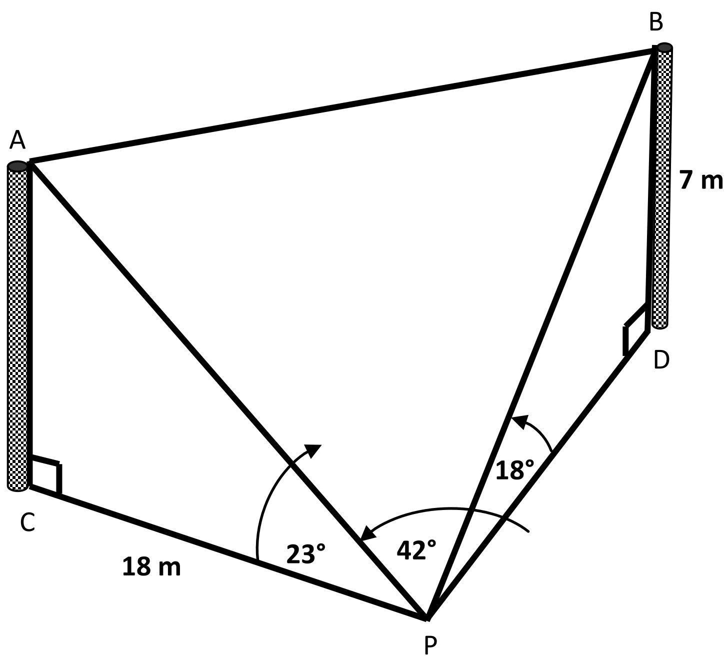
Thandi is standing at point P on the horizontal ground and observes two poles, AC and BD, of different heights. P, C and D are in the same horizontal plane. From P the angles of inclination to the top of the poles A and B are 23° and 18° respectively. Thandi is 18 m from the base of pole AC. The height of pole BD is 7 m.

Calculate, correct to TWO decimal places:

3.1 The distance from Thandi to the top of pole BD. (2)

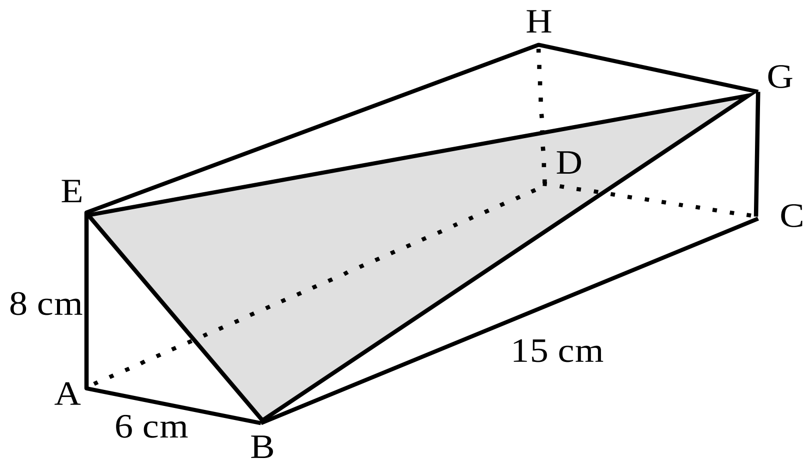
3.2 The distance from Thandi to the top of pole AC. (2)

3.3 The distance between the tops of the poles, that is the length of AB, if $A\hat{P}B = 42°$ (3)



Question 4

A rectangular block of wood has a breadth of 6 metres, height of 8 metres and a length of 15 metres. A plane cut is made through the block as shown in the diagram revealing the triangular plane that has been formed. Calculate the size of $E\hat{B}G$. (5)



Section C: Solutions

1.1	$AC^2 = (12m)^2 + (20m)^2 - 2(12m)(20m)\cos 110°$ $\therefore AC^2 = 708{,}1696688$ $\therefore AC = 26{,}6m$	✓ substitution into cosine rule ✓ answer (2)
1.2	$\dfrac{\sin B\hat{A}C}{12m} = \dfrac{\sin 110°}{26{,}6m}$ $\therefore \sin B\hat{A}C = \dfrac{12 \times \sin 110°}{26{,}6m}$ $\therefore \sin B\hat{A}C = 0{,}4239214831$ $\therefore B\hat{A}C = 25°$ **OR** $(12m)^2 = (20m)^2 + (26{,}6m)^2 - 2(20m)(26{,}6m)\cos B\hat{A}C$ $\therefore 1064\cos B\hat{A}C = 963{,}56m^2$ $\therefore \cos B\hat{A}C = 0{,}9056015038$ $\therefore B\hat{A}C = 25°$	✓ substitution into sine or cosine rule ✓ answer (2)
1.3	$(26{,}6m)^2 = (7m)^2 + (28m)^2 - 2(7m)(28m)\cos\hat{D}$ $\therefore 392\cos\hat{D} = 125{,}44$ $\therefore \cos\hat{D} = 0{,}32$ $\therefore \hat{D} = 71°$	✓ substitution into cosine rule ✓ $\cos\hat{D} = 0{,}32$ ✓ answer (3)

1.4	Area ABCD $= \frac{1}{2}(12m)(20m)\sin 110° + \frac{1}{2}(7m)(28m)\sin 71°$ $= 205,4m^2$	✓ $\frac{1}{2}(12m)(20m)\sin 110°$ ✓ $\frac{1}{2}(7m)(28m)\sin 71°$ ✓ answer (3)
2.1	In Δ ADC: $\hat{D} = 65°$ ($\angle$s of Δ) $\frac{AC}{\sin 65°} = \frac{158}{\sin 25°}$ $\therefore AC.\sin 25° = 158.\sin 65°$ $\therefore AC = \frac{158.\sin 65°}{\sin 25°}$ $\therefore AC = 338,83\text{m}$	✓ $\hat{D} = 65°$ ✓ $\frac{AC}{\sin 65°} = \frac{158}{\sin 25°}$ ✓ $AC = 338,83\text{m}$ (3)
2.2	In Δ ACB: $BC^2 = 338,83^2 + 1500^2 - 2(338,83)(1500)\cos 30°$ $\therefore BC^2 = 1\,484\,499,606$ $\therefore BC = 1218,4\text{m}$ In Δ DCB: $\tan\theta = \frac{DC}{BC}$ $\therefore \tan\theta = \frac{158}{1218,4}$ $\therefore \theta = 7,39°$	✓ cosine rule to get BC ✓ $BC = 1218,4\text{m}$ ✓ $\tan\theta = \frac{DC}{BC}$ ✓ $\tan\theta = \frac{158}{1218,4}$ ✓ $\theta = 7,39°$ (5)
2.3	Area $\Delta ABC = \frac{1}{2}(338,83)(1500)\sin 30°$ $\therefore$ Area $\Delta ABC = 127061,25m^2$	✓ area rule ✓ answer (2)
2.4	$AD^2 = (338,83)^2 + (158)^2$ $\therefore AD^2 = 139769,7689$ $\therefore AD = 373,86m$ $BD^2 = (1218,4)^2 + (158)^2$ $\therefore BD^2 = 1509462,56$ $\therefore BD = 1228,60m$ $(1500)^2 = (373,86)^2 + (1228,60)^2 - 2(373,86)(1228,60)\cos A\hat{D}B$ $\therefore 2(373,86)(1228,60)\cos A\hat{D}B = (373,86)^2 + (1228,60)^2 - (1500)^2$ $\therefore 918648,792\cos A\hat{D}B = -600770,7404$ $\therefore \cos A\hat{D}B = -0,6539721661$ $\therefore A\hat{D}B = 130,84°$	✓ Pythagoras ✓ $AD = 373,86m$ ✓ $BD = 1228,60m$ ✓ cosine rule ✓ substitution ✓ answer (6)

3.1	$\frac{7}{PB}=\sin 18°$ $\therefore PB=\frac{7}{\sin 18°}$ $\therefore PB=22{,}65247584..$	✓ definition ✓ answer (2)
3.2	$\frac{18}{PA}=\cos 23°$ $\therefore PA=\frac{18}{\cos 23°}$ $\therefore PA=19{,}55448679....$	✓ definition ✓ answer (2)
3.3	$AB^2=(22,65)^2+(19,55)^2-2(22,65)(19,55).\cos 42°$ $\therefore AB^2=237,0847954...$ $\therefore AB=15,40\text{ m}$	✓ cosine rule ✓ substitution ✓ answer (3)
4	In Δ AEB: $EB^2=8^2+6^2$ $\therefore EB^2=100$ $\therefore EB=10$ In Δ GBC: $BC^2=15^2+8^2$ $\therefore BC^2=289$ $\therefore BC=17$ In Δ ACB: $EG^2=15^2+6^2$ $\therefore EG^2=261$ $\therefore EG=\sqrt{261}$ In Δ EGB: $\therefore\left(\sqrt{261}\right)^2=17^2+10^2-\left(2(17)(10)\cos E\hat{B}G\right)$ $\therefore 261=389-\left(340\cos E\hat{B}G\right)$ $\therefore -128=-340\cos E\hat{B}G$ $\therefore \frac{32}{85}=\cos E\hat{B}G$ $\therefore E\hat{B}G=67{,}88°$	✓ EB ✓ BC ✓ EG ✓ cosine rule ✓ answer (5)