MORE TRIGONOMETRY 20 APRIL 2015

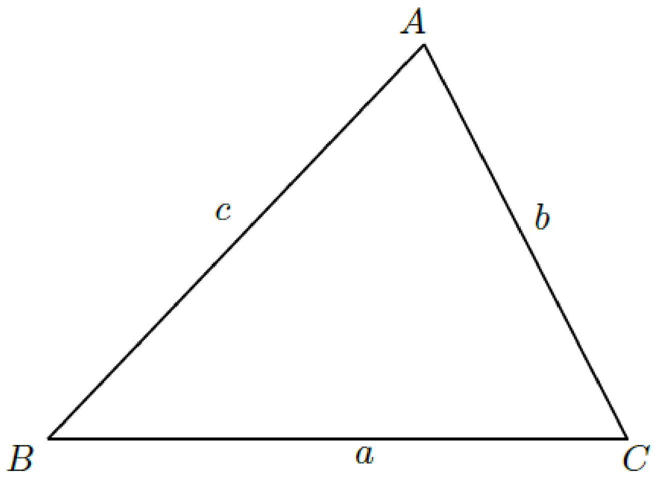
Section A: Summary Notes

Solving two-dimensional problems using the sine, cosine and area rules

- The **sine-rule** is used when the following is known in a triangle which is not a right angled triangle:

  - 2 angles and a side

  - 2 sides and an angle (not included)



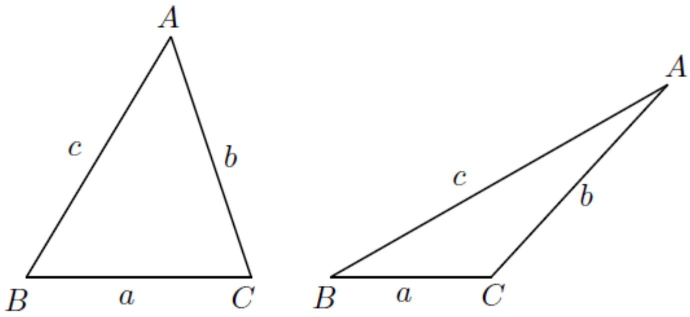
$$\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b} = \frac{\sin \hat{C}}{c}$$

$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

- The **cosine-rule** is used when the following is known in a triangle which is not a right angled triangle:

  - 3 sides

  - 2 sides and an included angle

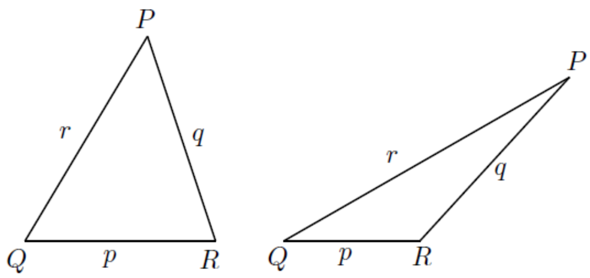


$$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$$

$$b^2 = a^2 + c^2 - 2ac\cos\hat{B}$$

$$c^2 = a^2 + b^2 - 2ab\cos\hat{C}$$

- The **area** of any triangle can be found when at least two sides and an included angle are known



$$\begin{aligned}\text{Area } \triangle PQR &= \frac{1}{2}qr\sin\hat{P}\\ &= \frac{1}{2}pr\sin\hat{Q}\\ &= \frac{1}{2}pq\sin\hat{R}\end{aligned}$$

Section B: Exam practice questions

Question 1

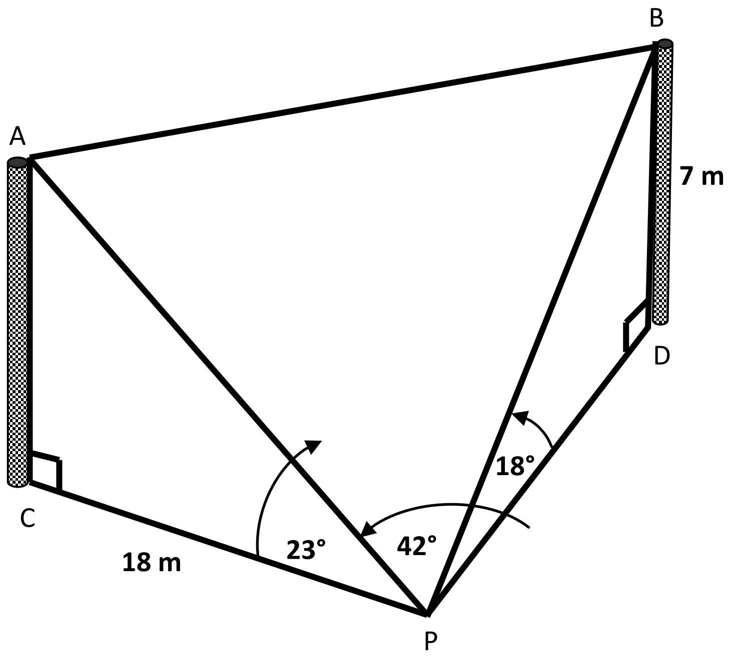
Thandi is standing at point P on the horizontal ground and observes two poles, AC and BD, of different heights. P, C and D are in the same horizontal plane. From P the angles of inclination to the top of the poles A and B are 23° and 18° respectively. Thandi is 18 m from the base of pole AC. The height of pole BD is 7 m.

Calculate, correct to TWO decimal places:

1.1 The distance from Thandi to the top of pole BD. (2)

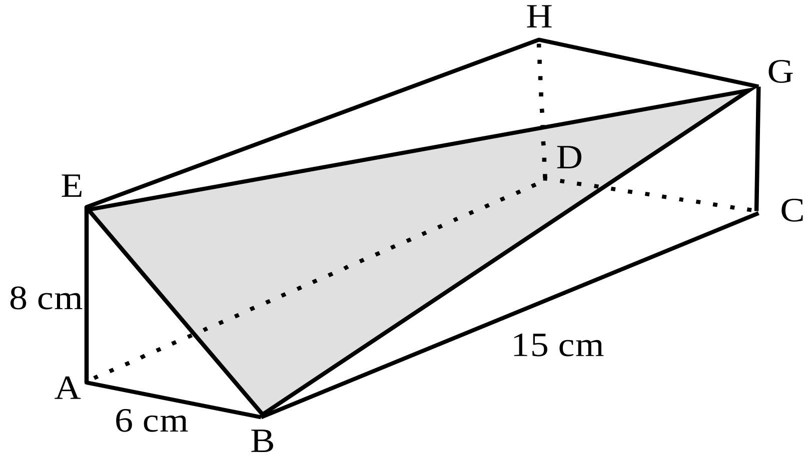
1.2 The distance from Thandi to the top of pole AC. (2)

1.3 The distance between the tops of the poles, that is the length of AB, if $A\hat{P}B = 42°$ (3)



Question 2

A rectangular block of wood has a breadth of 6 metres, height of 8 metres and a length of 15 metres. A plane cut is made through the block as shown in the diagram revealing the triangular plane that has been formed. Calculate the size of $E\hat{B}G$. (5)



Question 3

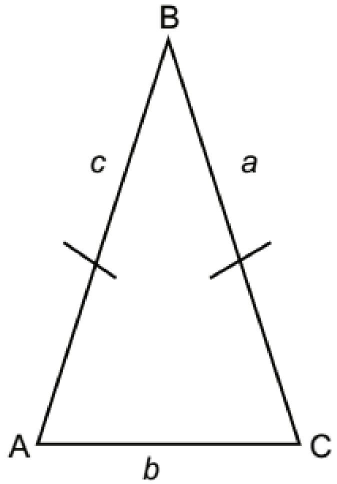
If $sin\,18° = t$ determine the following in terms of t.

a.) $cos\,18°$ (4)

b.) $sin\,78°$ (5)

Question 4

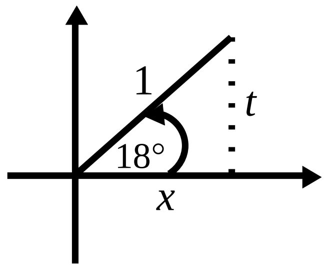
ΔABC is an isosceles triangle with AB = BC and AB = c, AC = b and BC = a



Prove that $cos\ B = 1 - \frac{b^2}{2a^2}$

Section C: Solutions

1.1	$\frac{7}{PB} = \sin 18°$ $\therefore PB = \frac{7}{\sin 18°}$ $\therefore PB = 22{,}65247584..$	✓ definition ✓ answer (2)
1.2	$\frac{18}{PA} = \cos 23°$ $\therefore PA = \frac{18}{\cos 23°}$ $\therefore PA = 19{,}55448679....$	✓ definition ✓ answer (2)
1.3	$AB^2 = (22{,}65)^2 + (19{,}55)^2 - 2(22{,}65)(19{,}55).\cos 42°$ $\therefore AB^2 = 237{,}0847954...$ $\therefore AB = 15{,}40\text{ m}$	✓ cosine rule ✓ substitution ✓ answer (3)
2	In Δ AEB: $EB^2 = 8^2 + 6^2$ $\therefore EB^2 = 100$ $\therefore EB = 10$ In Δ GBC: $BC^2 = 15^2 + 8^2$ $\therefore BC^2 = 289$ $\therefore BC = 17$ In Δ ACB: $EG^2 = 15^2 + 6^2$ $\therefore EG^2 = 261$ $\therefore EG = \sqrt{261}$ In Δ EGB: $\therefore \left(\sqrt{261}\right)^2 = 17^2 + 10^2 - \left(2(17)(10)\cos E\hat{B}G\right)$ $\therefore 261 = 389 - \left(340\cos E\hat{B}G\right)$ $\therefore -128 = -340\cos E\hat{B}G$ $\therefore \frac{32}{85} = \cos E\hat{B}G$ $\therefore E\hat{B}G = 67{,}88°$	✓ EB ✓ BC ✓ EG ✓ cosine rule ✓ answer (5)

3a.)	$\sin 18^\circ = t = \frac{t}{1}$ $x^2 = r^2 - y^2$ $\therefore x^2 = 1^2 - t^2$ $\therefore x^2 = 1 - t^2$ $\therefore x = \sqrt{1-t^2}$  $x^2 = r^2 - y^2$ $\therefore x^2 = 1^2 - t^2$ $\therefore x^2 = 1 - t^2$ $\therefore x = \sqrt{1-t^2}$ $\cos 18^\circ = \frac{\sqrt{1-t^2}}{1} = \sqrt{1-t^2}$	✓ diagram ✓ Pythagoras ✓ $x = \sqrt{1-t^2}$ ✓ $\cos 18^\circ = \frac{\sqrt{1-t^2}}{1} = \sqrt{1-t^2}$ (4)
3b.)	$\sin 78^\circ$ $= \sin(60^\circ + 18^\circ)$ $= \sin 60^\circ . \cos 18^\circ + \cos 60^\circ . \sin 18^\circ$ $= \left(\frac{\sqrt{3}}{2}\right)\sqrt{1-t^2} + \left(\frac{1}{2}\right)t$ $= \frac{\sqrt{3}\sqrt{1-t^2}}{2} + \frac{t}{2}$ $= \frac{\sqrt{3(1-t^2)} + t}{2}$	✓ $\sin(60^\circ + 18^\circ)$ ✓ $\sin 60^\circ . \cos 18^\circ + \cos 60^\circ . \sin 18^\circ$ ✓ $\left(\frac{\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2}\right)$ ✓ $\sqrt{1-t^2}$ and t ✓ $\left(\frac{\sqrt{3}}{2}\right)\sqrt{1-t^2} + \left(\frac{1}{2}\right)t$ (5)
4	$b^2 = a^2 + c^2 - 2ac.\cos B$ (cosine Rule) $b^2 = 2a^2 - 2a^2.\cos B$ $a = c$ (given) $b^2 = 2a^2(1 - \cos B)$ $\frac{b^2}{2a^2} = 1 - \cos B$ $\therefore \cos B = 1 - \frac{b^2}{2a^2}$	