FUNCTIONS & INVERSES

17 FEBRUARY 2014

Lesson Description

In this lesson we:

- Revise definition and terminology used to describe functions
- Define the inverse of a function

Summary

The **domain** is the set of *x*-values which the function can work with while the **range** is the set of *y*-values the function can produce.

Definition of a Function:

A function *f* is a relation between two variables such that for each element of the domain of *f*, there exists only one element of the range.

Types of Functions:

There are two kinds of function:

1. <u>One-to-one Function</u>:

   Each *x* is mapped to only one *y*, each *y* is mapped to only one *x*

   E.g. linear, hyperbolic, exponential and (some) cubic functions

2. <u>Many-to-one Function</u>:

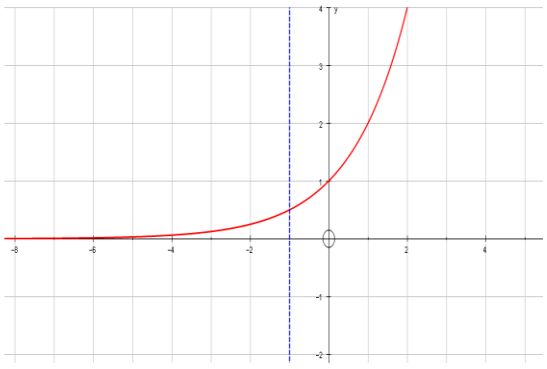
   Each *x* is mapped to only one *y* but a *y* may be mapped to more than one *x*.

   E.g. quadratic or (some) cubic functions.

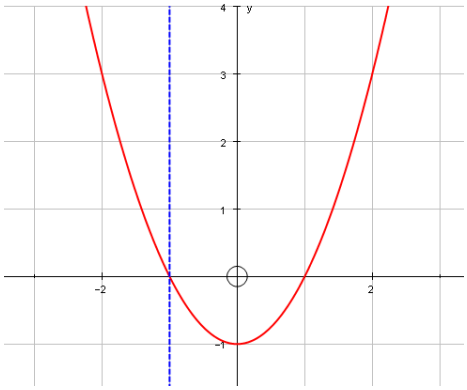
Vertical Line Test

To see if a graph represents a function use the 'vertical line test': if a vertical line cuts a graph in more than one place at any point, then the graph does not represent a function but a relation

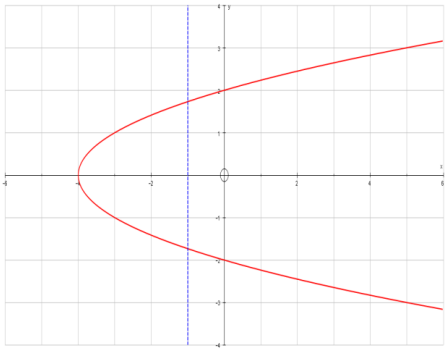
One-to-one function



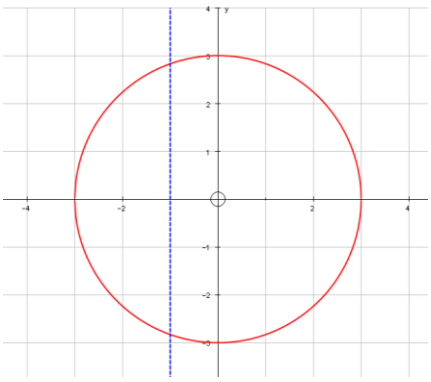
Many-to-one function



One-to-many non-function



Many-to-many non-function



Inverse of a function

An inverse of a function is a mapping of all the output values to the input values. The inverse of a function may not be a function. It is a reflection about the line y = x

Test Yourself

Question 1

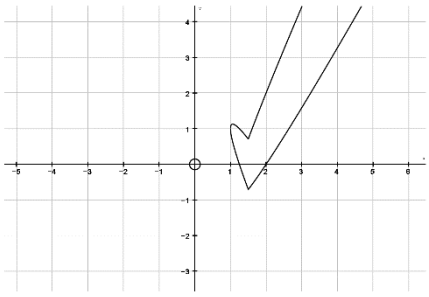
Say whether or not each of the following represents function or not:

(a) $y = \frac{3}{x-8} + 9$ (1)

(b) A graph which includes the following points:

$(-3;4)$ $(2;2)$ $(-2;2)$ $(7;7)$ *and* $(3;-5)$ (1)

(c) The following graph: (1)



Question 2

Consider the functions $f(x) = 2x + 4$ *and* $g(x) = x^2 + 2x$

Determine:

(a) $f(-2)$ (1)

(b) A *if* $f(A) = 0$ (2)

(c) $x \quad if \quad f(x) = g(x)$ (3)

(d) Give the domain of the function *g*. (2)

(e) Give the range of the function g by first writing it in the form

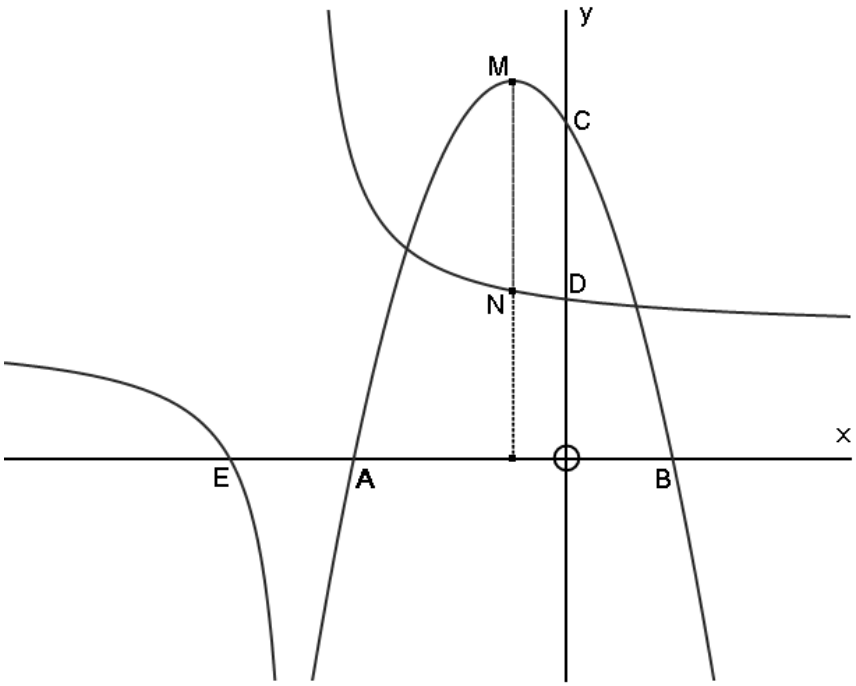
$g(x) = (x-p)^2 + q$. (4)

Improve your Skills

Question 1

The figure below represents the graphs of the following functions

$f(x) = -2x^2 - 4x + 16$ and $g(x) = \frac{8}{x+5} + 6$



a.) Determine the coordinates of A, B, C, D, E (6)

b.) If M is the turning point of $f(x)$, determine the length of MN (4)

c.) How many points of intersection do *f* and *g* share? (1)

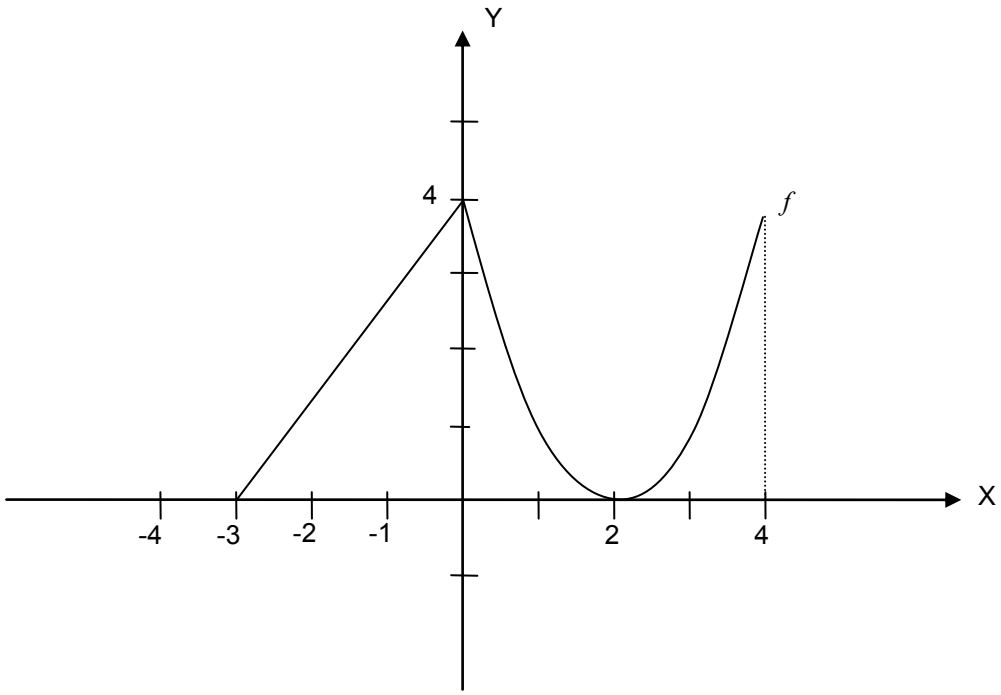
d.) State the equations of the vertical and horizontal asymptotes of *g*(*x*) (2)

e.) Give the range of *f* (2)

f.) Is the inverse of *g* a function? Explain (2)

Question 2

Study the sketch below, then answer the questions that follow:



a.) Describe the domain of f. (2)

b.) Is f^{-1} a function or a non-function? Explain. (2)

c.) For the straight line f, when $x \in [-3;0]$, write down the equation of $f^{-1}(x)$, in the form:

$f^{-1}(x) =$ ..... (2)

Question 3

The graphs of $h(x) = 4^{x+p} + q$ and $g(x) = ax^2 + bx + c$ are drawn below.

K(4 ; 10) is a common point of the two graphs. A(–1 ; 0) and B(2 ; 0) are the *x*-intercepts of the graph of $g(x)$.

Use the information about the two graphs to determine the values of p, q, a, b and c.