

EXPONENTIAL & LOGARITHMIC FUNCTIONS

03 MARCH 2014



Lesson Description

In this lesson we:

- Investigate the relationship between exponential functions and their inverses.



Summary

Terminology

A function is a mathematical rule that maps an input value to a unique output value.

The **domain** of a function is the set of all input values

The **range** of a function is the set of all output values

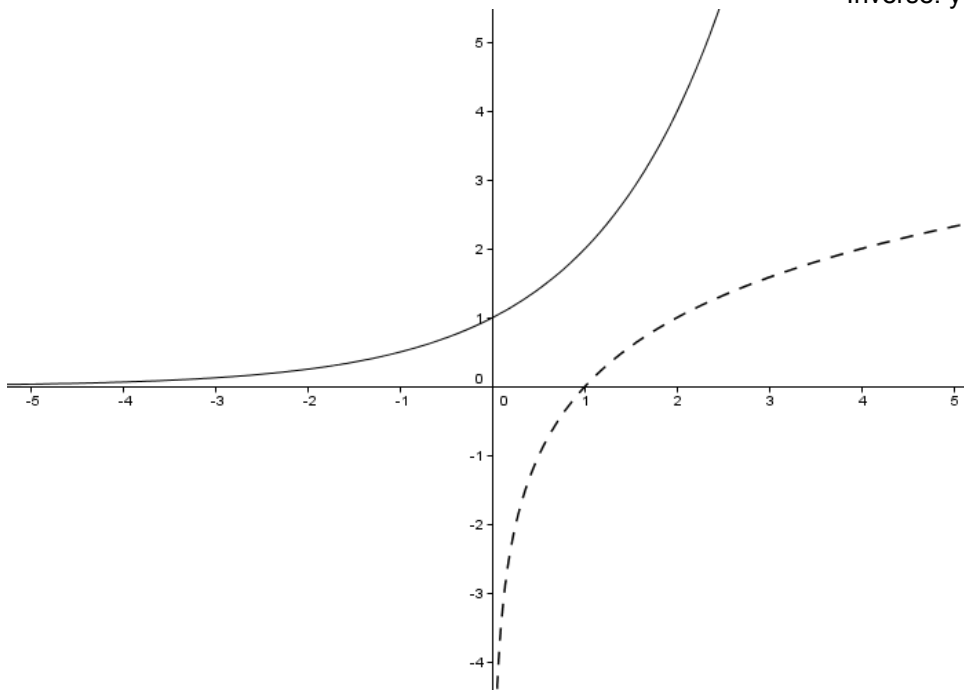
Definition of logarithm

Graphs of Exponential Function and the Logarithm

Exponential Function

$y = 2^x$

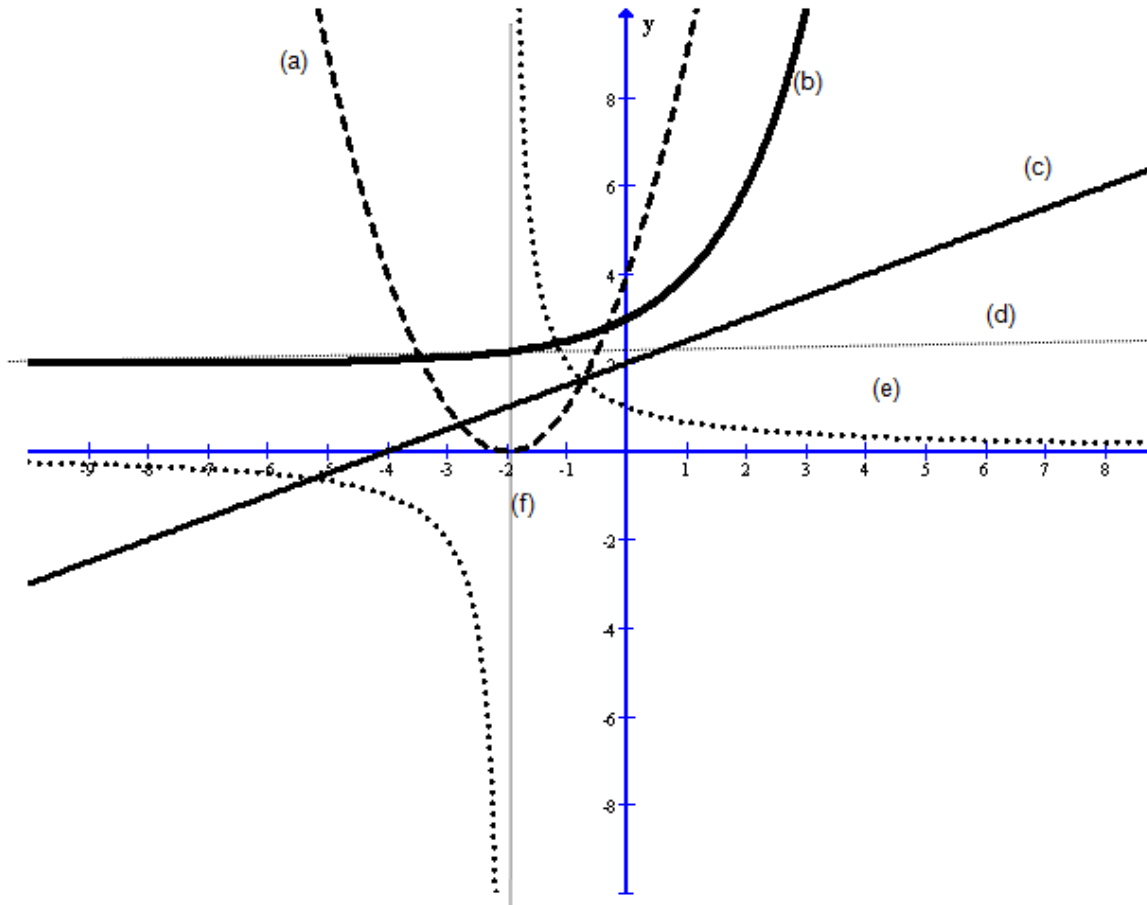
Inverse: $y = \log_2 x$





Test Yourself

Choose the equation, from the list below, which best describes each of the following graphs. Write ONLY the number of the equation next to the letter of the graph.



- | | | | |
|-------------------------|-------------------------|---------------------------|---------------------------|
| 1. $y = x + 2$ | 2. $y = x - 2$ | 3. $y = \frac{x}{2} + 2$ | 4. $y = \frac{x}{2} - 2$ |
| 5. $y = (x + 2)^2$ | 6. $y = (x - 2)^2$ | 7. $y = x^2 + 2$ | 8. $y = -x^2 + 2$ |
| 9. $y = 2$ | 10. $y = -2$ | 11. $x = 2$ | 12. $x = -2$ |
| 13. $y = 2x$ | 14. $y = -2x$ | 15. $y = \frac{2}{x} + 2$ | 16. $y = \frac{2}{x} - 2$ |
| 17. $y = \frac{2}{x+2}$ | 18. $y = \frac{2}{x-2}$ | 19. $y = 2^x + 2$ | 20. $y = 2^x - 2$ |
| 21. $y = 2^{x+2}$ | 22. $y = 2^{x-2}$ | | |

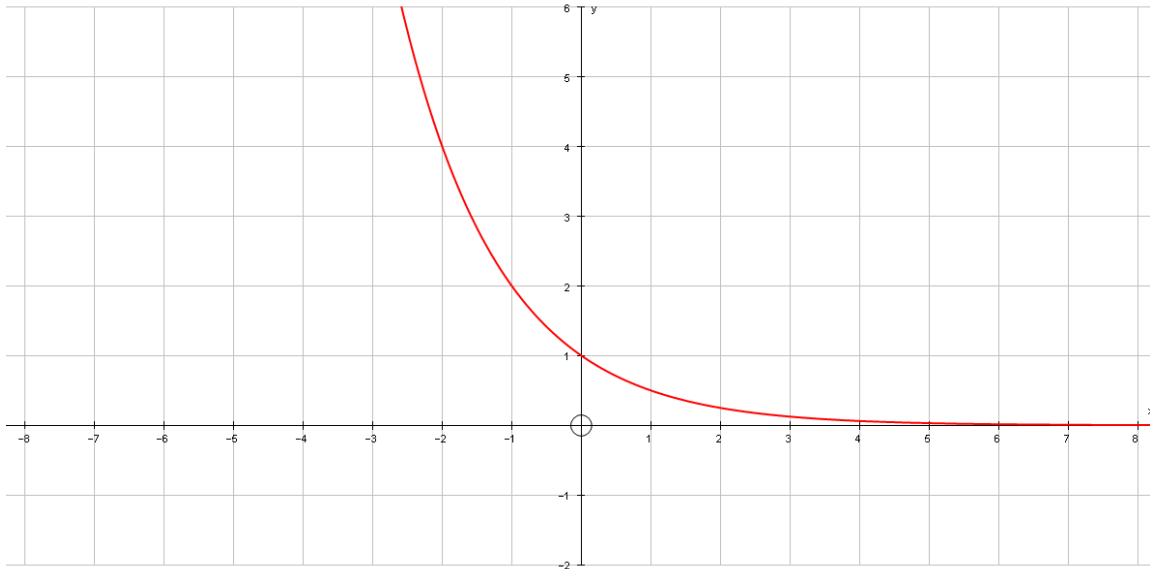
a _____; b _____; c _____; d _____; e _____; f _____ (12)



Improve your Skills

Question 1

The figure shows the graph of $g(x) = a^x$.



- Determine the value of a .
- Give the domain and range of g .
- Draw the graph g^{-1} , the inverse of g .
- Does g^{-1} represent a function? Explain.
- Give the equation of g^{-1} in the form $y = \dots$

Question 2

2.1. Given: $f(x) = 2^x - 8$.

Determine:

- an equation of h if $h(x) = f(2x) + 8$
- an equation of h^{-1} in the form $y = \dots$

2.2. Given $f(x) = 2^x$, $g(x) = f(x - 2)$, $h(x) = f^{-1}(x)$.

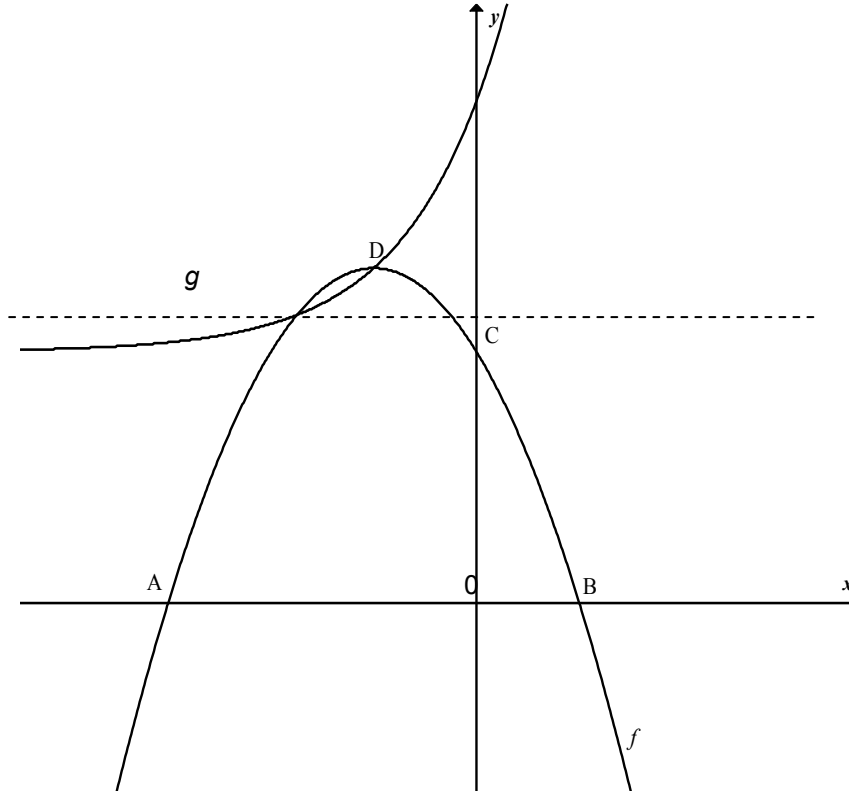
Write down the equations of g and h in the form $y = \dots$

Question 3

The graphs of $f(x) = -(x+1)^2 + 4$ and $g(x) = a \cdot 3^x + q$ are sketched below.

A and B are the x-intercepts of f .

C is the y-intercept of f and lies on the asymptote of g . The two graphs intersect in D, the turning point of f .

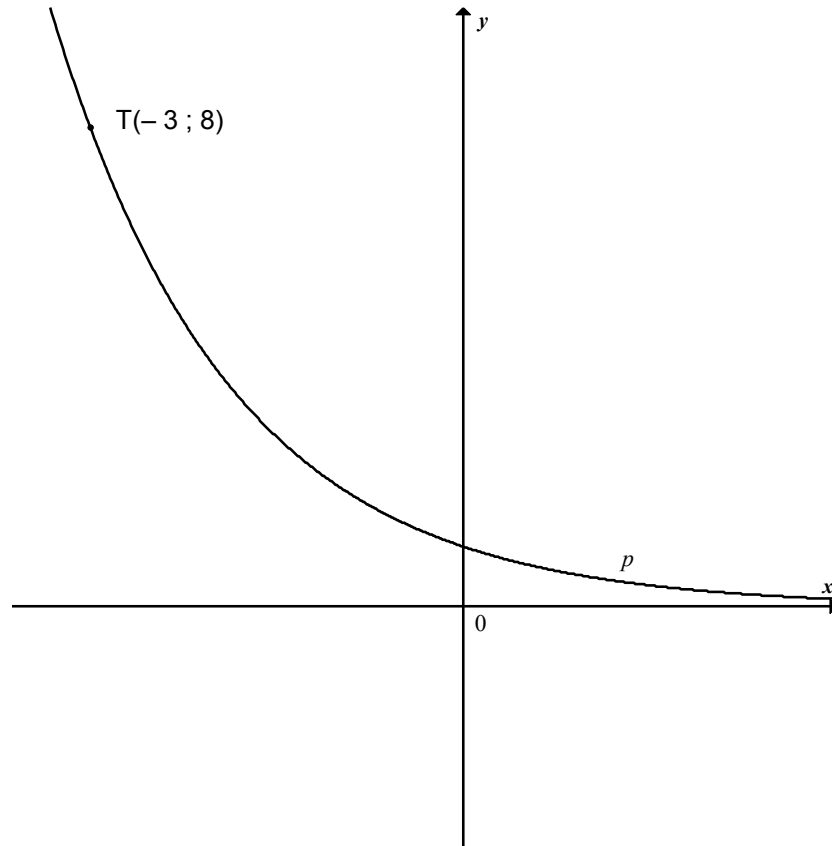


- Determine the coordinates of A and B. (4)
- Determine the equation of g. (3)
- Calculate the coordinates of the point on f for which the tangent to f will have a gradient of 1. (4)
- Write down the values of k for which $f(x) - k$ will always be a negative value. (2)

[13]

Question 4

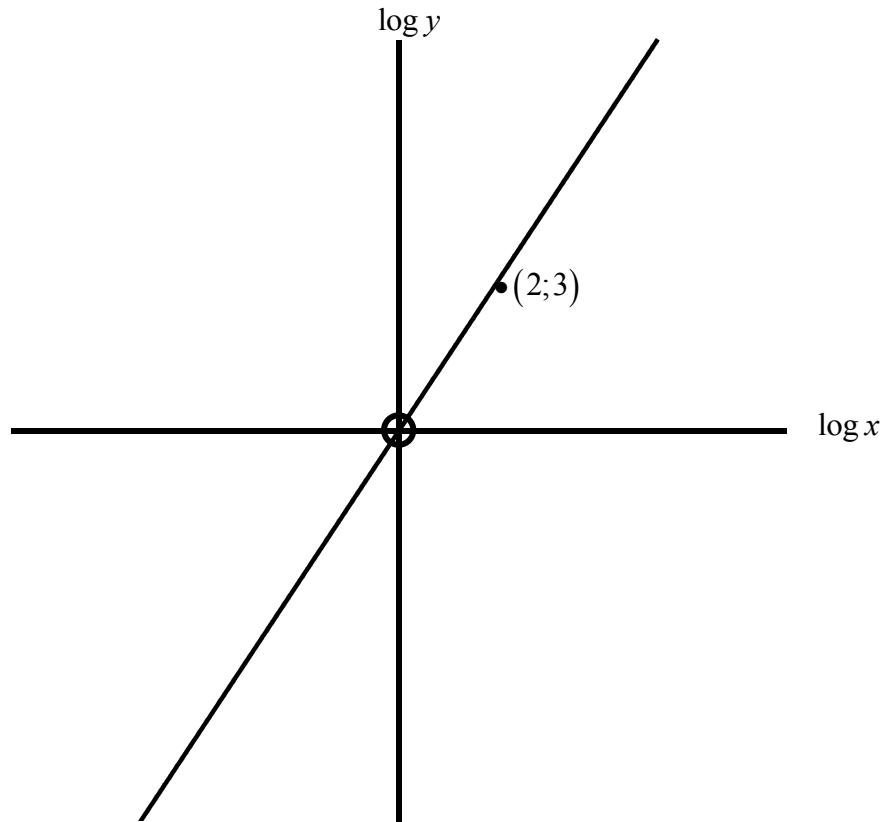
The graph of $p(x) = a^x$ is sketched below. The point $T(-3; 8)$ lies on the graph of p .



- a.) Calculate the value of a . (3)
- b.) Write down the equation of $p^{-1}(x)$ in the form $y = \dots$ (2)
- c.) For which values of x will $p^{-1}(x) > -3$? (2)
- d.) Write down the equation of q if q is the result of p shifted 3 units to the right. (2)

Question 5

The figure below shows the linear relationship between $\log y$ and $\log x$



- a.) Write down $\log y$ in terms of $\log x$ (2)
 b.) Hence, write down y in terms of x (2)