REVIEW JUNE EXAM QUESTIONS 21 JULY 2014

Lesson Description

In this lesson we:

- Revise concepts from both Paper One and Two.

Summary

Paper One

Number Patterns and Sequences

- Arithmetic and Geometric Sequences
- Arithmetic and Geometric Series

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1}\ ;\ r \neq 1 \qquad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$$

Functions, Inverses and Logarithms

- Revision of gr 11 functions (Parabola, Hyperbola, Exponential function)
- Inverse graphs (Notation, restricting Domain and Range)
- Logarithmic function and Laws of Logarithms

Financial Mathematics

- Gr 11 revision and use of logarithms
- Future and Present Value Annuities

$$A = P(1 + ni) \qquad A = P(1 - ni)$$

$$A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$F = x\left[\frac{(1+i)^n - 1}{i}\right] \qquad P = x\left[\frac{1 - (1+i)^{-n}}{i}\right]$$

Algebra and Differential Calculus

- Remainder and Factor Theorem
- Limits
- Average gradient
- Differentiation
- Tangents
- Cubic Graphs
- Optimisation

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Probability

- Gr 11 revision
- Fundamental counting principle (factorial notation, dealing with repetition, selections)

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Paper Two

Analytical Geometry

- Gr 11 Revision
- Equation of a circle
- Intersection of circles
- Tangents to circles

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

Trigonometry

- Gr 11 revision
- Compound Angles
- Double Angles
- Identities using compound and double angles
- Trig equations
- Two and three dimensional Trigonometry

$$In\ \ \Delta ABC: \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x\ ;\ y) \to (x\cos\theta + y\sin\theta\ ;\ y\cos\theta - x\sin\theta) \qquad (x\ ;\ y) \to (x\cos\theta - y\sin\theta\ ;\ y\cos\theta + x\sin\theta)$$

Euclidean Geometry

- Gr 11 Circle Geometry Revision
- Ratios and Proportion
- Similarity

Statistics

- Gr 11 Revsion
- Scatter Plots (line of regression, least squares regression line, correlation coefficient)

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

Test Yourself

Question 1

Which of the following statements are true:

i) 1, -1, 1, -1, … is a geometric sequence

ii) The first six elements in the set of consecutive triangular numbers form an arithmetic sequence.

iii) The elements in any set of consecutive natural numbers from an arithmetic sequence.

A. Only A
B. Only B
C. Only C
D. Only A and B
E. Only A and C

Question 2

Which of the following is/are true?

i) $(0{,}01)^{-x} = 10^{2x}$

ii) the 5th term of a geometric sequence is 36 and the 10th term is 1 152. The common ratio r is thus 2 and the first term is $\frac{9}{4}$.

iii) The solution of $log_2(x-3) + log_2(x-4) - 1 = 0$ is $x = 2\ \ or\ \ x = 5$

A. Only A

B. Only B

C. Only C

D. Only A and B

E. Only B and C

Question 3

The solution of $2^{2x} - 3.2^x - 4 = 0$ is

A. $x = 2$

B. $x = -1\ \ or\ \ x = 4$

C. $x = 0\ \ or\ \ x = 2$

D. $x = 1\ \ or\ \ x = -4$

E. $x = 0$

Question 4

A triangle ABC has vertices: $A1, 3\ ,\ \ B1; -2\ \ and\ C{-4}\ ; -2.$

Which of the following is true:

i) $d(B, C) = \sqrt{41}$

ii) $\hat{C} = 90°$

iii) The midpoint of the hypotenuse is $(\frac{-3}{2}; \frac{1}{2})$

A. Only A

B. Only B

C. Only C

D. Only A and C

E. Only B and C

Question 5

Which of the following statements is/are true?

i) $y = -\frac{1}{2}x$+3 does not define a one-to-one function

ii) $x^2 + y^2 = 25$ does not define a one-to-one function

iii) $y = 2x^2 + 7x + 3$ does not define a one-to-one function

A. Only A

B. Only B

C. Only C

D. Only A and B

E. Only B and C

Question 6

The circle with equation $x^2 + y^2 - 2x + 4y = 0$ has centre C and radius r, where

A. C(1;-2) ; $r = \sqrt{5}$

B. $C(-1; 2); r = \sqrt{5}$

C. $C(1; -4); r = \sqrt{17}$

D. $C(-1; 4); r = 17$

E. $C(1; -2); r = 5$

Question 7

The general equation of a line through (4 ; 0) which is parallel to the line containing points (1 ; 0) and (3 ; -1) is :

A. $y = 2x + 2$

B. $x + 2y - 4 = 0$

C. $y = 2(x - 4)$

D. $2x - y + 8 = 0$

E. $2x + y - 8 = 0$

Question 8

Which of the following statements is/are true?

i) $(2x + 3y)^2 = (2x)^2 + (3y)^2$

ii) $(-a)^{-x} = a^x$

iii) $a^{-\frac{1}{2}} + b^{-\frac{1}{2}} = \frac{\sqrt{a}+\sqrt{b}}{\sqrt{ab}}$

A. Only A

B. Ony B

C. Only C

D. A, B and C

E. None of them

Question 9

Determine: $D_x[\frac{x^3-1+x^2}{x}]$

A. $3x^2 + 2x - \frac{x}{2}$

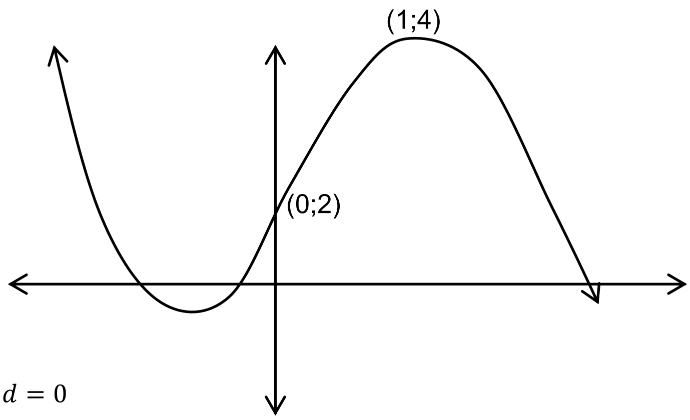
B. $2x + \frac{x}{2} + 1$

C. $3x^2 + 2x$

D. $2x^2 + 2x$

Question 10

If polynomial $f(x) = -x^3 + bx^2 + cx + d$ whose graph is shown below has a y-intercept of (0;2) and a turning point at (1;4), determine b, c and d.



A. $b = 3, c = 2, d = 0$

B. $b = 0, c = 3, d = 2$

C. $b = 2, c = 0, d = 3$

D. $b = -3, c = 0, d = -2$

Improve your Skills

Question 1(a)

1; 3; 5 are the first three terms of the first differences of a quadratic sequence.

The 7th term of the quadratic sequence is 35.

i) Determine the 6th and 5th terms of the quadratic sequence.

ii) Determine the nth term of the quadratic sequence.

Question 1(b)

Calculate the value of n if:

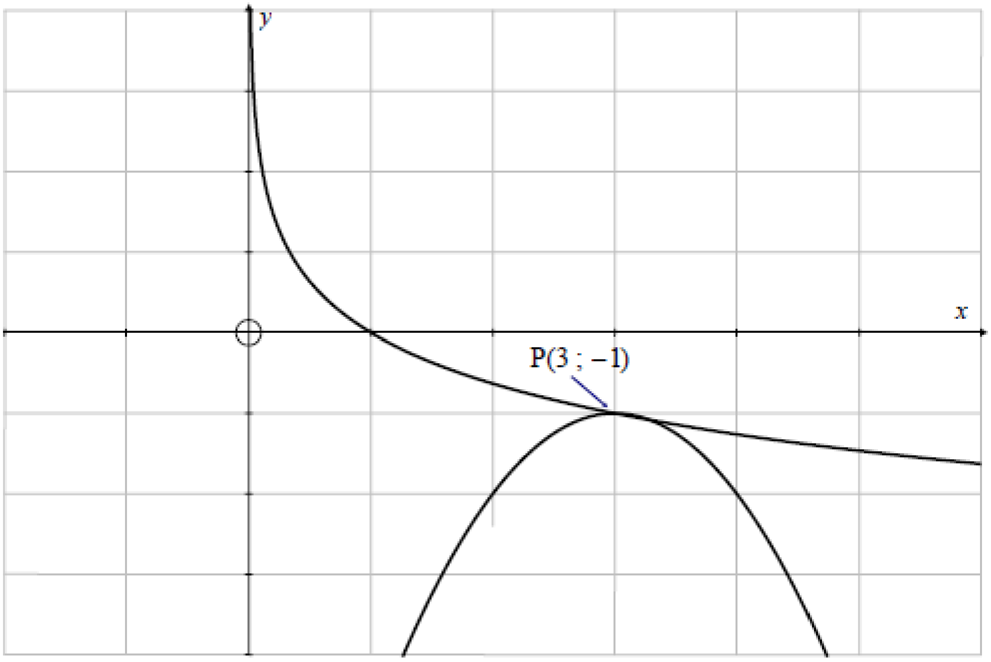
$$\sum_{k=1}^{n} 2(3)^{k-1} = 531440$$

Question 2

Refer to the figure:

The graphs of $y = g(x) = log_a x$ and $y = h(x) = -(x-3)^2$ are given.

The point $P(3\,;\,-1)$ lies on the graph of both g and h.



Determine:

i) the vlaue of a

ii) the equation which defines $g^{-1}(x)$ in the form $y = \cdots$

iii) the x –values for which $1 \leq g^{-1}(x) \leq 3$

iv) a possible restriction that could be placed on $h(x)$ to ensure that $h^{-1}(x)$ is a function.

v) the values of x for which $g(x).h(x) < 0$.

Question 3

Calculate the derivatice of the following:

i) $x^2(1 - \frac{1}{x})$

ii) $h(x) = \frac{\sqrt[3]{x^2} - 3x}{\sqrt{x}}$

Question 4

A crate used on vegetable farms in the Ponono Area is the form of a rectangular prism which is open on top. It has a volume of 1 cubic meter. The length and breadth of its base is $2x$ and x meters respectively. The height is h meters. The material used to manufacture the base of this container costs R200 per square meter and for the sides, R120 per square meter.

i) Express h in terms of x

ii) Show that the cost, C, of the material is given by:

$C(x) = 400x^2 + 360x^{-1}$

iii) Calculate the value of x for which the cost of the material will be a minimum and hence the minimum cost of the material.

Question 5

Show without the use of a calculator that:

i) $sin15° = \frac{\sqrt{6}-\sqrt{2}}{4}$

ii) $\frac{\tan(180°+\theta).\cos(360°-\theta)}{\sin(180°-\theta).\cos(90°+\theta)+\cos(540°+\theta).\cos(-\theta)}$

iii) Show: $\frac{\sin 33°}{sin11°} - \frac{cos33}{cos11°} = 2$

iv) Determine the general solution of the equation:

$\frac{tan3x}{tan24°} - 1=0$