WORK-ENERGY THEOREM

21 APRIL 2015

Section A: Summary Notes

Work

Work is done on an object when the object moves in the same plane as the force. If a force is applied to an object at 90^{o} to the motion of the object then that force does no work on the object. When a force is applied at an angle (θ) to an object then then work is done by the component of the force that causes the object to move. This is given by the following equation:

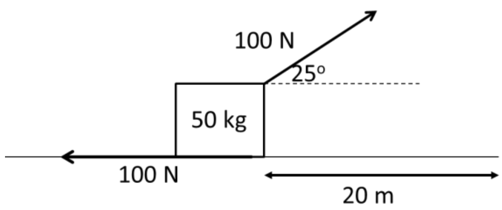
$W = F\Delta x \cos\theta$	W = work done (J) F = force applied (N) Δx = displacement (m) θ = angle of force to the horizontal

Work is a scalar quantity therefore it has magnitude but not direction. Work can be a negative value when the energy of the system decreases and positive when the energy of the system increases.

<u>Net work</u> on an object can be calculated by applying the definition of work to each force acting on the object while it is being displaced and then adding up each contribution.

Example 1

Calculate the net work done on a box of 50 kg which is being pulled by a 100 N at an angle of 25^{0} to the horizontal for 20 m while experiencing a frictional force of 10 N.



Solution

F = 100 N

θ = 25^{o}

f = 10 N

Δx = 20 m

$$W_F = F\Delta x \cos\theta$$
$$= (100)(20)(\cos 25°)$$
$$= 1812{,}62\ J$$

$$W_f = f\Delta x \cos\theta$$
$$= (10)(20)(\cos 180°)$$
$$= -200\ J$$

Friction always acts at 180^{o} to the direction of motion

$$W_{net} = W_F + W_f$$
$$= 1812{,}62 + (-200)$$
$$= 1612{,}62\ J$$

The net work done on an object can also be calculated by first calculating the net force acting on the object and then applying the equation.

The **work – energy theorem** states that the net work done on an object by the net force is equal to the change in kinetic energy of the object.

$$W_{net} = E_{kf} - E_{ki}$$

If there are non-conservative forces, e.g. friction or air resistance present then the system is not closed and the mechanical energy of the system will change. Mechanical energy is the sum of the potential energy and kinetic energy of the object. Although mechanical energy is not conserved the total energy of the system is still conserved.

$W_{nc} = \Delta E_k + \Delta E_p$	W_{nc} = work done by non-conservative force (J) ΔE_k = change in kinetic energy (J) ΔE_p = change in potential energy (J)
Remember: $E_k = \frac{1}{2}mv^2$ $E_p = mgh$	E_k = kinetic energy (J) m = mass (kg) v = velocity ($m.s^{-1}$) E_p = potential energy (J) g = acceleration due to gravity (9,8 $m.s^{-2}$) h = height (m)

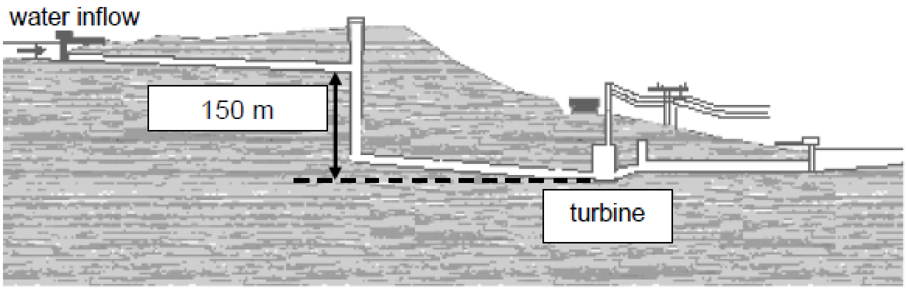
Example 2

(Taken from DoE Paper 1 Nov. 2008)

The diagram below represents how water is funnelled into a pipe and directed to aturbine at a hydro-electric power plant. The force of the falling water rotates theturbine. Each second, 200 m^3 of water is funnelled down a vertical shaft to the turbine below. The vertical height through which the water falls upon reaching the turbine is 150 m.

Ignore the effects of friction.

NOTE: One m^3 of water has a mass of 1 000 kg.



1. Calculate the mass of water that enters the turbine each second.

Solution:

m = (200)(1000)

= 2 x 10^5 kg

2. Calculate the kinetic energy of this mass of water when entering the turbine.

Use the work- energy theorem.

Solution

F = mg *Applied force is equal to the weight of the water.*

= (2 x 10^5)(9,8)

= 1,96 x 10^6 N

$$W_{net} = \Delta E_k$$
$$F\Delta x \cos\theta = E_{kf} - E_{ki}$$
$$(1{,}96 \times 10^6)(150)(\cos 0°) = E_{kf} - 0$$
$$\therefore E_{kf} = 2{,}94 \times 10^8\ J$$

E_{ki} = 0J as the water is considered to be stationary before being pumped up the hill

3. Calculate the maximum speed at which this mass of water enters the turbine.

Solution

$$E_{kf} = \frac{1}{2}mv_f^2$$
$$2{,}94 \times 10^8 = \frac{1}{2}(2 \times 10^5)v_f^2$$
$$v_f^2 = 2{,}94 \times 10^3$$
$$\therefore v_f = 54{,}22\ m.s^{-1}$$

Energy

Law of conservation of energy states that energy cannot be created or destroy but can be transferred from one form to another.

Mechanical energy is the sum of the kinetic energy and potential energy of an object.

$E_{mech} = E_p + E_k$	E_{mech} = mechanical energy (J) E_p = potential energy (J) E_k = kinetic energy (J)

In an isolated system (no friction or air resistance) mechanical energy is always conserved.

$$E_{mech\ initial} = E_{mech\ final}$$
$$(E_p + E_k)_{initial} = (E_p + E_k)_{final}$$

Power

Power is the rate at which work is done.

$P = \frac{W}{\Delta t}$	P = power (W) W = work (J) Δt = time (s)

When an object is moving at a constant speed then the force being applied to the object is constant. The average power can then be calculated using the following equation:

$P_{av} = Fv_{av}$	P_{av} = average power (W) F = applied force (N) v_{av} = speed ($m.s^{-1}$)

Example 3

Consider the water being pumped up the hill as described in example 2.

1. Assume that a generator converts 85% of the maximum kinetic energy gained by the water into hydro-electricity. Calculate the electrical power output of the generator

Solution

$$P = \frac{W}{\Delta t} \times \frac{85}{100}$$ W = E_{kf} = 2,94 x 10^8 J

$$= \frac{2{,}94 \times 10^8}{1} \times 0{,}85$$

$$= 2{,}50 \times 10^8\ W$$

2. Explain what happens to the 15% of the kinetic energy that is NOT convertedinto electrical energy.

Solution

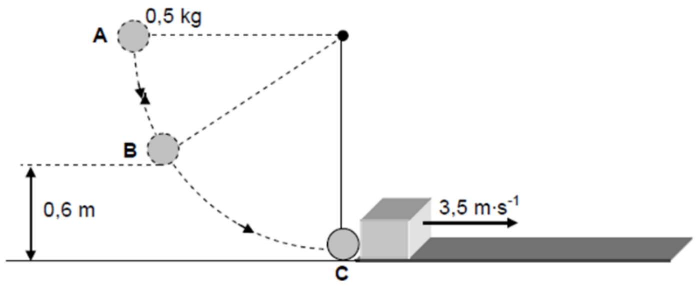
The kinetic energy is converted into sound or heat in the turbine.

Section B: Practice Questions

Question 1

(Taken from NCS November Paper I 2010)

A steel ball of mass 0,5 kg is suspended from a string of negligible mass. It is released from rest at point A, as shown in the sketch below. As it passes through point B, which is 0,6 m above the ground, the magnitude of its velocity is 3 m.s^{-1}. (Ignore the effects of friction)



1.1. Write the principle of the conservation of mechanical energy in words. (2)

1.2. Calculate the mechanical energy of the steel ball at point B. (4)

As the steel ball swings through its lowest position at point C, it collides with a stationary crate of mass 0,1 kg. Immediately after the collision, the crate moves at a velocity of 3,5 m.s^{-1} to the right.

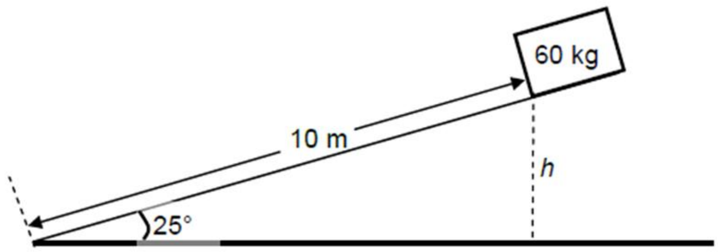
1.3. Calculate the velocity of the steel ball immediately after the collision. (7)

[13]

Question 2

(Taken from Feb – March 2010)

A box of mass 60 kg starts from rest at a height *h* and slides down a rough slope of length 10 m, which makes an angle of 25^o with the horizontal. It undergoes a constant acceleration of magnitude 2 m.s^{-2} while sliding down the slope.



2.1. State the work-energy theorem in words. (2)

2.2. Draw a free-body diagram to show ALL the forces acting on the cardboard box while it slides down the slope. (3)

2.3. The reaches the bottom of the slope

Calculate the following:

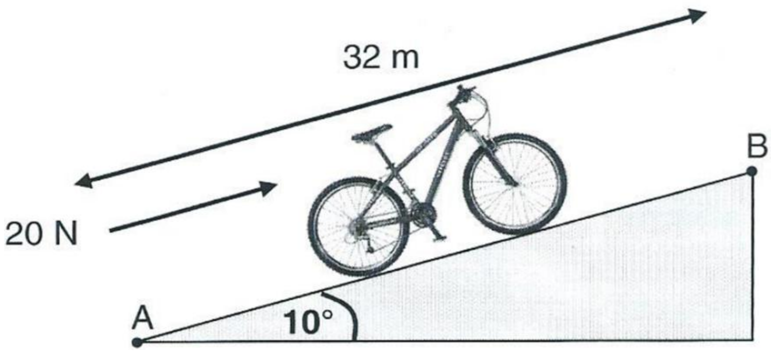
2.3.1. The kinetic energy of the box, using the equations of motion. (5)
2.3.2. The work done on the box by the gravitational force (4)
2.3.3. The work down on the box by the frictional force, using the work-energy theorem. (4)
2.3.4. The magnitude of the frictional force acting on the box. (3)

[21]

Question 3

(Taken from Gauteng Preparatory Exam Paper 1 2014)

A cyclist pushes his bicycle of mass 6,1 kg up an incline with a force of 20 N. The bicycle is pushed from an initial velocity of 5 $m.s^{-1}$ from point A to point B. The road is inclined at 10^o to the horizontal and the distance from A to B is 32 m as show below.



The road surface exerts a force of friction of 11 N on the bicycle tyres.

3.1. Calculate the work done by the cyclist on the bicycle. (3)
3.2. Use the work-energy theorem and calculate the magnitude of the velocity of the bicycle at 32 m. (5)
3.3. Explain why friction forces are regarded as non-conservative forces. (2)

[10]

Section C: Solutions

Question 1

1.1. The total mechanical energy is conserved ✓in an isolated system. ✓ (2)

1.2.

$$E_{mech} = E_p + E_K ✓$$

$$= mgh + \frac{1}{2}mv^2$$

$$= (0,5)(9,8)(0,6)✓ + \frac{1}{2}(0,5)(3)^2✓$$

$$= 5,19\ J✓$$

(4)

1.3. *1st – Calculate the steel ball's speed before it hits the crate*

$$(E_P + E_K)top = (E_P + E_K)bottom✓$$

$$5,19 = 0 + \frac{1}{2}mv^2$$

$$5,19✓ = 0 + \frac{1}{2}(0,5)v^2✓$$

$$\therefore v^2 = 20,76$$

$$\therefore v = 4,56\ m.s^{-1}$$

2nd – Now apply law of conservation of momentum principle

$$\Sigma p_i = \Sigma p_i✓$$

$$m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$$

$$(0,5)(4,56) + 0✓ = (0,5)v_{f1} + (0,1)(3,5)✓$$

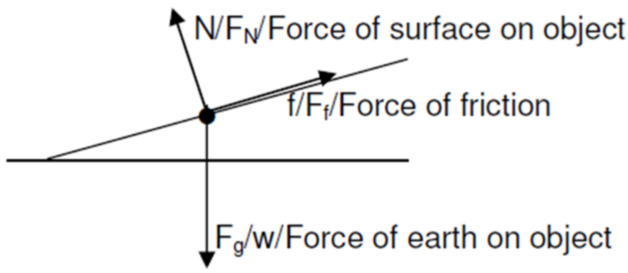
$$\therefore v_{f1} = 3,86\ m.s^{-1}✓ \text{to the right}$$

(7)

Question 2

2.1. The net work done on an object is equal to the change in the object's kinetic energy✓✓ (2)

2.2.



(3)

2.3.1.

$$v_f^2 = v_i^2 + 2a\Delta x\ ✓$$

$$= (0)^2 + (2)(2)(10)✓$$

$$= 40$$

$$\therefore v_f = 6,32\ m.s^{-1}$$

$$E_K = \frac{1}{2}mv_f^2✓$$

$$= \frac{1}{2}(60)(40)✓$$

$$= 1\ 200\ J\ ✓$$

(5)

2.3.2. $W_g = F_{g\parallel}\Delta x\cos\theta$ ✓

$= [mg\sin\theta]$✓$\Delta x\cos\theta$

$= (60)(9{,}8)(\sin 25°)(10)(\cos 0°)$✓

$= 2\ 485\ J$ ✓ (4)

2.3.3. $W_{net} = \Delta E_K$✓

$W_g + W_f = E_{Kf} - E_{Ki}$✓

$2\ 485 + W_f = 1\ 200 - 0$ ✓

$\therefore W_f = -1\ 285\ J$ ✓ (4)

2.3.4. $W_f = F_f\Delta x\cos\theta$ ✓

$-1\ 285 = F_f(10)(\cos 180°)$✓

$\therefore F_f = 128{,}5\ N$ ✓ (3)

Question 3

3.1. $W = F\Delta x\cos\theta$ ✓

$= (20)(32)(\cos 0°)$✓

$= 640\ J$ ✓ (3)

3.2. **Option 1**

$F_{net} = F - F_g\sin\theta - f$

$= 20 - (6{,}1)(9{,}8)(\sin 10°) - 11$✓

$= -1{,}38\ N$

$= 1{,}38\ N\ backwards$

Note:

If 'net' left out of F_{net}: max $\frac{2}{5}$

$W_{net} = \Delta E_K$✓

$F_{net}\Delta x\cos\theta = \frac{1}{2}m(v_f^2 - v_i^2)$

$(1{,}38)(32)(\cos 180°)$✓ $= \frac{1}{2}(6{,}1)(v_f^2 - 5^2)$✓

$\therefore v_f^2 = 10{,}51$

$\therefore v_f = 3{,}24\ m.s^{-1}$✓

Option 2

$W_{net} = \Delta E_k$

$F\Delta x\cos\theta + f\Delta x\cos\theta + F_g\Delta x\cos\theta = \frac{1}{2}m(v_f^2 - v_i^2)$✓

$[(20)(32)(\cos 0°) + (11)(32)\cos 180°]$✓ $+ (6{,}1)(9{,}8)(32)\cos 10°$ ✓ $= \frac{1}{2}(6{,}1)(v_f^2 - 5^2)$✓

$\therefore v_f^2 = 10{,}51$

$\therefore v_f = 3{,}24\ m.s^{-1}$✓

Option 3

$W_{net} = \Delta E_k$

$F\Delta x\cos\theta + f\Delta x\cos\theta + F_g\Delta x\cos\theta = \frac{1}{2}m(v_f^2 - v_i^2)$✓

$[(20)(32)(\cos 0°) + (11)(32)\cos 180°]$✓ $+ [(6,1)(9,8)\sin 10°](32)\cos 180°$ ✓

$= \frac{1}{2}(6,1)(v_f^2 - 5^2)$✓

$\therefore v_f^2 = 10,51$

$\therefore v_f = 3,24\ m.s^{-1}$✓ (5)

3.3. The net work done by the non-conservative force / frictional force depends on the path the object travelled ✓✓

OR

The mechanical energy is not constant ✓✓ for a non-conservative force (2)