DIFFERENTIAL CALCULUS

30 JUNE 2014

Checklist

Make sure you know how to:

- Calculate the average gradient of a curve using the formula $average\ gradient = \frac{f(x+h)-f(x)}{h}$
- Find the derivative by first principles using the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$
- Use the rules of differentiation to differentiate functions without going through the process of first principles.

Rules for Differentiation - Summary:

1: If $f(x) = k$, where k is a constant real number, then $f'(x) = 0$
2: If $f(x) = x^n$, where n is a constant real number, then $f'(x) = n\,x^{n-1}$
3: If $f(x) = k.g(x)$, where k is a constant real number, then $f'(x) = k\,.g'(x)$

(Note, when applying rules of differentiation always ensure brackets are multiplied out, surds are changed to exponential form and any terms with the variable in the denominator must be rewritten in the form $y = k.x^{-1}$ before the rule can be applied).

- Recognise the various ways to represent a function and its derivative

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Notation:

- $f(x) = 5x^2 + 3x$ $\rightarrow$ $f'(x) =$
- $y = 5x^2 + 3x$ $\rightarrow$ $\frac{dy}{dx} = \cdots$
- $D_x[5x^2 + 3x]$ $\rightarrow$ $=$

- Sketch a cubic graph from the standard equation of $y = ax^3 + bx^2 + cx + d$ by finding x-intercepts, y-intercept, stationary points and point of inflection
- Find the equation of a cubic graph when given the x −intercepts by using the formula $y = a(x - x_1)(x - x_2)(x - x_3)$
- Determine the equation of a tangent to a cubic function.
- Calculate the maximum or minimum value in a problem. In all maxima and minima problems you need to prove or derive a formula to represent the given scenario. You will then always need to calculate the value of the variable which will give you this maximum or minimum. They way in which this is done is to set the derivative of the function equal to zero.
- Draw and interpret the graph of the derivative function.

Exam Questions

Question 1

a.)   Determine $f'(x)$ from first principles if $f(x) = -3x^2 + x$

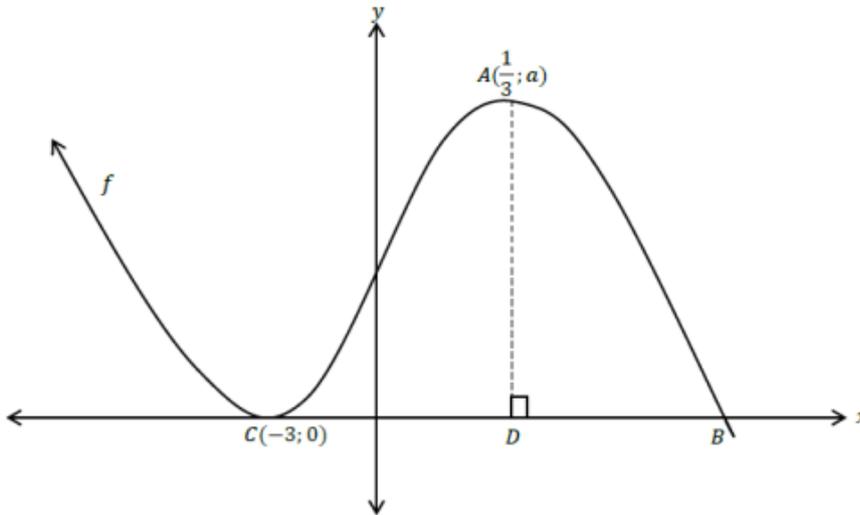b.)   Sketch the graph of $f(x) = -3x^2 + x$ and $f'(x)$ on the same set of axes.

Question 2

Find $\frac{dy}{dx}$ if $y = 8x^3 - 4\sqrt{x} + \frac{4x^2 + 3}{x}$; $x > 0$

Question 3

The gradient of the curve $y = -\frac{3}{4}x^2 + 7x - 4$ at a certain point is 1. Find the point.

Question 4

The above sketch represents the function $f: x \to (x + 3)^2(-x + t)$.

$A(\frac{1}{3}; a)$ and $C(-3 ; 0)$ are the turning points of f.

a)   Prove that $a = 2$

b)   Find the coordinates of B.

c)   Find a correct to one decimal place.

Question 5

The length of a wire 200 meters long, is cut into two pieces. One piece is used to form an equilateral triangle x units in length. The other piece is used to form a square.

a)   Show that the sides of the square are $50 - \frac{3}{4}x$ units in length.

b)   Determine the area of the square in terms of x.

c)   Determine the value of x so that the area of the square is a maximum.

Test Yourself

Question 1

Determine: $D_x\left[\frac{x^3-1+x^2}{x}\right]$

A $\quad 3x^2 + 2x - \frac{x}{2}$ B $\quad 2x + \frac{x}{2} + 1$ C $\quad 3x^2 + 2x$ D $\quad 2x^2 + 2x$

Question 2

The equation of the tangent to $f(x) = -x^2 + x + 2$ at point P(1 ; 2) is

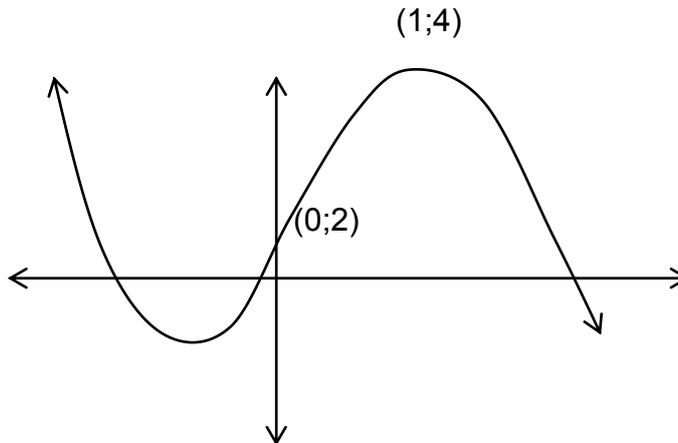A $\quad y = -\frac{1}{2}x + 2\frac{1}{2}$ B $\quad y = x - 1$ C $\quad y = -x + 1$ D $\quad y = -x + \frac{1}{2}$

Question 3

If $f(x) = -x^3 + 7x^2 - 15x + 9$, determine $f(x) = 0$

A $\quad$ (1 ; 0) and (3;0) B $\quad$ (1;0), (-1;0) and (3;0)

C $\quad$ (1;0), (3;0) and (-3;0) D $\quad$ (1;0), (3;0) and (5;0)

Question 4

If polynomial $f(x) = -x^3 + bx^2 + cx + d$ whose graph is shown below has a $\quad$ y-intercept of (0;2) and a turning point at (1;4), determine b, c and d.

(1;4)

(0;2)

A $\quad b = 3, c = 2, d = 0$ B $\quad b = 0, c = 3, d = 2$

C $\quad b = 2, c = 0, d = 3$ D $\quad b = -3, c = 0, d = -2$

Question 5

The point of inflection of the function $y = -2x^3 - 12x^2 - 18x$ is:

A $\quad$ (2;4) B $\quad$ (4 ; -2) C $\quad$ (2 ; 0) D $\quad$ (-2 ; 4)