EUCLIDEAN GEOMETRY: CIRCLES

Checklist

Make sure you learn proofs of the following theorems:

- The line drawn from the centre of a circle perpendicular to a chord bisects the chord
- The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle circumference (on the same side of the chord as the centre);
- The opposite angles of a cyclic quadrilateral are supplementary
- The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment

You must understand application of the following (these are corollaries derived from theorems or axioms):

- Angles in a semi-circle
- Equal chords subtend equal angles at the circumference
- Equal chords subtend equal angles at the centre
- In equal circles, equal chords subtend equal angles at the circumference
- In equal circles, equal chords subtend equal angles at the centre.
- The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.
- If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.
- Tangents drawn from a common point outside the circle are equal in length.

Knowledge of geometry from previous grades will be integrated into questions in the exam.

- Euclidean Geometry makes up ± 35% of Maths P2
- If you have attempted to answer a question more than once, make sure you cross out the answer you do not want marked, otherwise your first answer will be marked and the rest ignored.
- You must learn proofs of the theorems however proof of the converse of the theorems will not be examined.
Exam Questions

Question 1

1.1 Complete the following statement so that it is valid:
The angle subtended by an arc at the centre of a circle is...........

1.2 In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also NA = NC and \( \angle B = 38^\circ \)

1.2.1 Calculate with reasons:
   a) \( \angle KMA \)  
   b) \( \angle T2 \)
   c) \( \angle C \)
   d) \( \angle K4 \)

1.2.2) Show that NK=NT
Question 2

(Adapted from Nov. 2013, Paper 3, Question 10)

In the diagram points P, Q, R and T lie on the circumference of a circle. MW and TW are tangents to the circle at P and T respectively. PT is produced to meet RU at U.

\( \angle MPR = 75^\circ; \angle PQT = 29^\circ; \angle QTR = 34^\circ \)

Calculate the values of the angles labelled: a, b, c, d

(9)

Question 3

(Adapted from Feb/March 2014, Paper 3, Question 8)

In the diagram below, O is the centre of the circle. Chord AB is perpendicular to diameter DC

CM : MD = 4 : 9 and AB = 24 units.
3.1 Determine an expression for DC in terms of $x$ if CM = 4x units
3.2 Determine an expression for OM in terms of x.
3.3 Hence, or otherwise, calculate the length of the radius.

Question 4
In the given diagram, BME is a diameter of circle centre M and FE is a tangent at E. Secant FDA is drawn such that DE = DF. Chord AC cuts BE at G and MD cuts EC at H. $\angle F = x$

Prove that:
4.1 $\angle C_2 = 2\angle C_1$
4.2 GMHC is a cyclic quadrilateral

Test Yourself

Question 1
Choose the option that makes this statement valid: The angle between the tangent and chord...
A) is equal
B) is equal to the angle at the centre of the circle
C) is equal to the angle in the alternate segment
D) is 90°

Question 2
True or false, if false provide a reason for your answer: If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.

Question 3
Choose the option that makes this statement valid: Equal chords in equal circles subtend...
A) equal angles at the circumference of the circles
B) equal angles at the centre of the circles
C) unequal angles at the centre of the circles
D) unequal angles at the circumference of the circles

**Question 4**
True or false, if false provide a reason for your answer: If the opposite angles of a quadrilateral are complementary then the quadrilateral is cyclic.

**Question 5**
Choose the option that makes this statement valid: The perpendicular bisector of a chord...
A) bisects the tangent to the circle
B) bisects the chord
C) passes through the centre of the circle
D) forms an angle of 90° at the centre of the circle

**Additional Question**
(Adapted from Nov 2013, Paper 3, Question 12)
In the diagram below two circles intersect one another at D and B. AB is a straight line such that it intersects the circle BCD at point E. BC is a straight line such that it intersects the circle ABD at F. DE, DB and DF are joined.

\[ \angle F_2 = 180° - 2x \text{ and } FC = FD \]

Calculate, giving reasons, in terms of x:

1.1 \( \angle DEB \) (3)
1.2 \( \angle A \) (2)
1.3 Hence or otherwise prove that \( ED \parallel BC \) (3)