



## EUCLIDEAN GEOMETRY: CIRCLES

### Checklist

Make sure you learn proofs of the following theorems:

- The line drawn from the centre of a circle perpendicular to a chord bisects the chord
- The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle circumference (on the same side of the chord as the centre);
- The opposite angles of a cyclic quadrilateral are supplementary
- The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment

**You must understand application of the following (these are corollaries derived from theorems or axioms):**

- Angles in a semi-circle
- Equal chords subtend equal angles at the circumference
- Equal chords subtend equal angles at the centre
- In equal circles, equal chords subtend equal angles at the circumference
- In equal circles, equal chords subtend equal angles at the centre.
- The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.
- If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.
- Tangents drawn from a common point outside the circle are equal in length.

**Knowledge of geometry from previous grades will be integrated into questions in the exam.**

- Euclidean Geometry makes up  $\pm$  35% of Maths P2
- If you have attempted to answer a question more than once, make sure you cross out the answer you do not want marked, otherwise your first answer will be marked and the rest ignored.
- You must learn proofs of the theorems however proof of the converse of the theorems will not be examined.

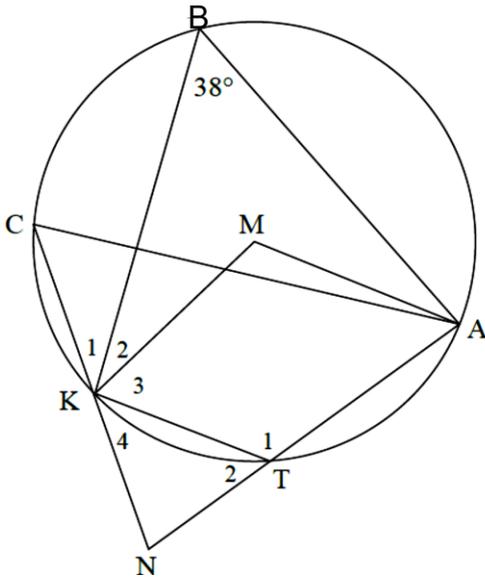




**Exam Questions**

**Question 1**

- 1.1 Complete the following statement so that it is valid:  
The angle subtended by an arc at the centre of a circle is.....
- 1.2 In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also  $NA = NC$  and  $\angle B = 38^\circ$



- 1.2.1 Calculate with reasons:
- |                 |     |
|-----------------|-----|
| a) $\angle KMA$ | (2) |
| b) $\angle T_2$ | (2) |
| c) $\angle C$   | (2) |
| d) $\angle K_4$ | (2) |
- 1.2.2) Show that  $NK=NT$  (2)





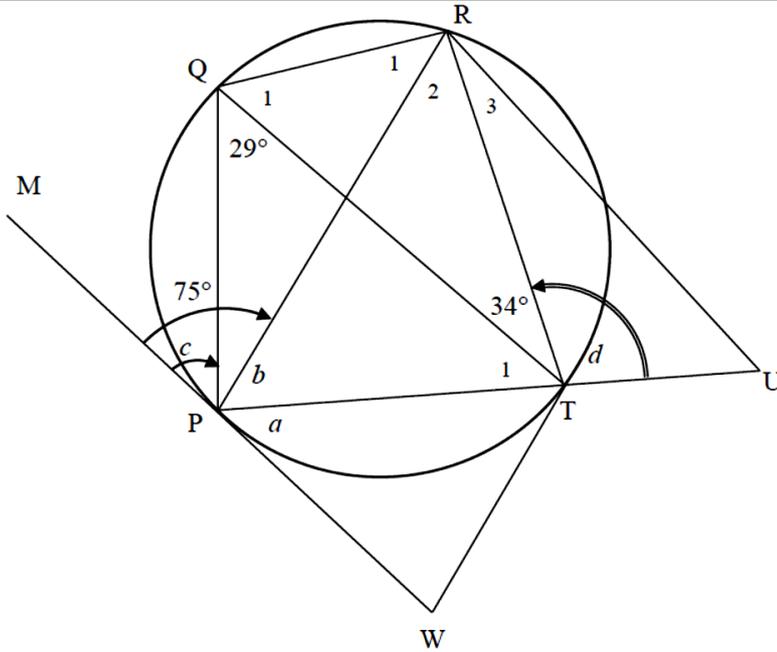
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**Question 2**

(Adapted from Nov. 2013, Paper 3, Question 10)

In the diagram points P, Q, R and T lie on the circumference of a circle. MW and TW are tangents to the circle at P and T respectively. PT is produced to meet RU at U.

$\angle MPR = 75^\circ$ ;  $\angle PQT = 29^\circ$ ;  $\angle QTR = 34^\circ$



Calculate the values of the angles labelled: a, b, c, d

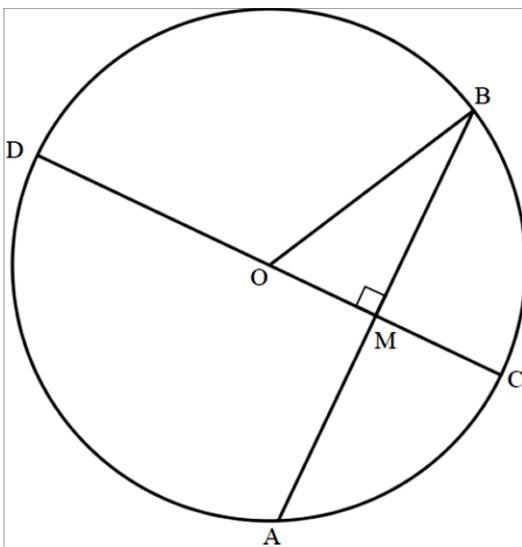
(9)

**Question 3**

(Adapted from Feb/March 2014, Paper 3, Question 8)

In the diagram below, O is the centre of the circle. Chord AB is perpendicular to diameter DC

CM : MD = 4 : 9 and AB = 24 units.





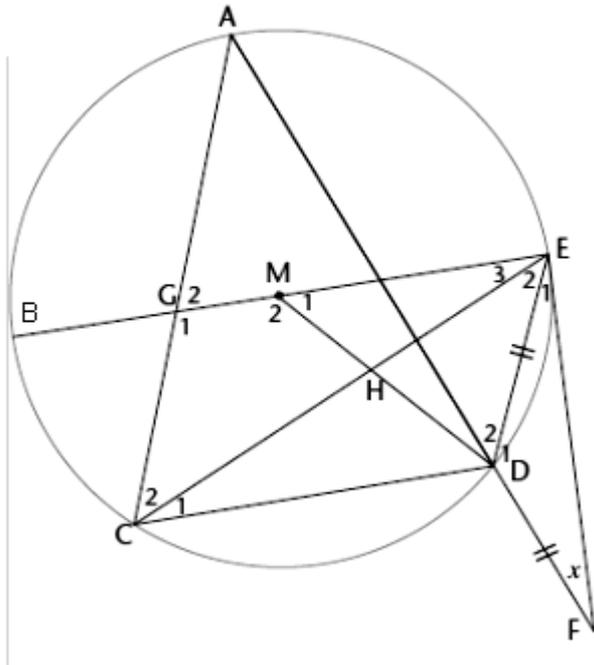
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- 3.1 Determine an expression for DC in terms of x if  $CM = 4x$  units (1)
- 3.2 Determine an expression for OM in terms of x. (2)
- 3.3 Hence, or otherwise, calculate the length of the radius. (4)

**Question 4**

In the given diagram, BME is a diameter of circle centre M and FE is a tangent at E. Secant FDA is drawn such that  $DE = DF$ . Chord AC cuts BE at G and MD cuts EC at H.  $\angle F = x$



**Prove that:**

- 4.1  $\angle C2 = 2\angle C1$  (9)
- 4.2 GMHC is a cyclic quadrilateral (5)

**Test Yourself**

**Question 1**

Choose the option that makes this statement valid: The angle between the tangent and chord...

- A) is equal
- B) is equal to the angle at the centre of the circle
- C) is equal to the angle in the alternate segment
- D) is  $90^\circ$

**Question 2**

True or false, if false provide a reason for your answer: If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.

**Question 3**

Choose the option that makes this statement valid: Equal chords in equal circles subtend...

- A) equal angles at the circumference of the circles
- B) equal angles at the centre of the circles





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**Grade 12**

- C) unequal angles at the centre of the circles
- D) unequal angles at the circumference of the circles

**Question 4**

True or false, if false provide a reason for your answer: If the opposite angles of a quadrilateral are complementary then the quadrilateral is cyclic.

**Question 5**

Choose the option that makes this statement valid: The perpendicular bisector of a chord...

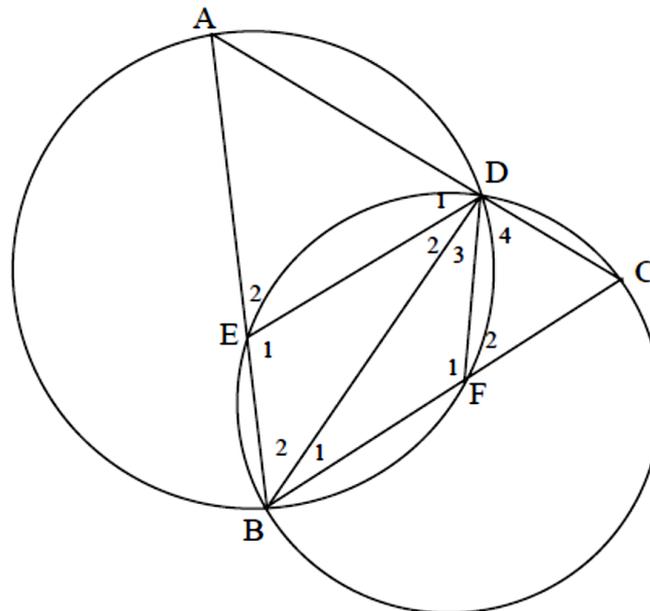
- A) bisects the tangent to the circle
- B) bisects the chord
- C) passes through the centre of the circle
- D) forms an angle of  $90^\circ$  at the centre of the circle

**Additional Question**

*(Adapted from Nov 2013, Paper 3, Question 12)*

In the diagram below two circles intersect one another at D and B. AB is a straight line such that it intersects the circle BCD at point E. BC is a straight line such that it intersects the circle ABD at point F. DE, DB and DF are joined.

$\angle F_2 = 180^\circ - 2x$  and  $FC = FD$



Calculate, giving reasons, in terms of x:

- 1.1  $\angle DEB$  (3)
- 1.2  $\angle A$  (2)
- 1.3 Hence or otherwise prove that  $ED \parallel BC$  (3)

