A Guide to Using Probability

Teaching Approach

It is very important to revise Grade 10 concepts with your pupils as they will need this basic foundation to build their Grade 11 and 12 probability knowledge on. These can be very interactive lessons where you can use die, bag of coloured marbles, cards or many other devices to illustrate how probability works. Before starting this section ask learners to give you everyday examples where probability is used.

It works well to first explain the difference between independent and dependent events and then in later lessons solve probability problems with the aid of Venn and tree diagrams or contingency tables. The idea of conditional probabilities are introduced but we do not go into much detail or examples using the conditional probability formulae.

If pupils recall their Grade 10 work quite well then the diagrams may be used to assist in explaining simple problems involving independent and dependent events. It is important to stress to learners that they must not confuse mutually exclusive events with independent events.

When dealing with Venn diagrams make sure learners start with the intersections and then work outward! These can become very messy and confusing if they are approached incorrectly.

Tree diagrams are not always the best choice to solve simple probability problems however for dependent events this is the way to go.

Although two-way contingency tables are covered in Grade 12 we have introduced the concept to Grade 11 learners in this series as these are present in a number of Grade 11 textbooks that learners may be exposed to.

These videos should be watched in the order they are presented in. The concepts within probability build on each other. The task video contains questions that cover all skills and should be watched at the end of the section.
Video Summaries

Some videos have a ‘PAUSE’ moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch of show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next day's lesson; if desired, learners can be given specific questions to answer in preparation for the next day’s lesson

1. Revision of Probability
   This video covers revision of Grade 10 concepts; Venn diagrams, the rule for mutually exclusive events i.e. \(P(A \text{ or } B) = P(A) + P(B)\), the rule for complementary events i.e. \(P(\text{not } A) = 1 - P(A)\) and the identity \(P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)\).

2. Independent and Dependent Events
   This video shows how to identify independent and dependent events and apply the product rule for independent events i.e. \(P(A \text{ and } B) = P(A) \times P(B)\). It also mentions the converse of the product rule.

3. Using Venn Diagrams
   We start with a brief revision of Venn diagrams and then explain how they are used to solve probability questions.

4. Using Tree Diagrams
   This video uses tree diagrams for the probability of consecutive or simultaneous events which are not necessarily independent. Independent and dependent events are also covered.

5. Using Contingency Tables
   This video looks at setting up a contingency table and working out if events are independent from the table.
# Resource Material

<table>
<thead>
<tr>
<th>1. Revision of Probability</th>
<th><a href="https://everythingmaths.co.za/grade-10/10-probability/10-probability-08.cnxmlplus">https://everythingmaths.co.za/grade-10/10-probability/10-probability-08.cnxmlplus</a></th>
<th>A summary note on Grade 10 Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Independent and Dependent Events</td>
<td><a href="http://www.mathsisfun.com/data/probability-events-independent.html">http://www.mathsisfun.com/data/probability-events-independent.html</a></td>
<td>Independent events- this explains probability concepts in a clear and concise manner and has examples that can be used in class.</td>
</tr>
<tr>
<td></td>
<td><a href="http://www.youtube.com/watch?v=RWPu4TZguVs">http://www.youtube.com/watch?v=RWPu4TZguVs</a></td>
<td>Youtube video showing how to work out dependent and independent events.</td>
</tr>
<tr>
<td>3. Using Venn Diagrams</td>
<td><a href="http://www.purplemath.com/modules/venndiag4.htm">http://www.purplemath.com/modules/venndiag4.htm</a></td>
<td>This site demonstrates how to solve word problems that involve Venn diagrams</td>
</tr>
<tr>
<td></td>
<td><a href="http://www.easycalculation.com/statistics/probability-tree.php">http://www.easycalculation.com/statistics/probability-tree.php</a></td>
<td>Probability tree calculator: Probability tree allows us to generate and list all the events under one chart. Select the number of main events, branch events and then enter a label and a probability for each event.</td>
</tr>
<tr>
<td>5. Using Contingency Tables</td>
<td><a href="http://www.youtube.com/watch?v=vBIIw7HXIY">http://www.youtube.com/watch?v=vBIIw7HXIY</a></td>
<td>YouTube video that goes into the basics of contingency tables.</td>
</tr>
<tr>
<td></td>
<td><a href="http://cnx.org/content/m16835/latest/">http://cnx.org/content/m16835/latest/</a></td>
<td>Problems that require contingency tables.</td>
</tr>
</tbody>
</table>
Task

Question 1
A letter is chosen at random from the word RANDOM. What is the probability that the letter is:
1.1 D
1.2 a vowel
1.3 not a vowel

Question 2
State whether the following events are independent, mutually exclusive or neither:
2.1 Getting a head when tossing a coin and getting a six when rolling a dice.
2.2 Drawing a heart from a regular pack of 52 cards on the first draw and drawing a heart from the same pack on a second draw without replacing the first card.
2.3 Choosing the name of a boy or choosing the name of a girl when one name is chosen at random from a list of all learners in a class of boys and girls.

Question 3
A fair coin is tossed three times:
3.1 What is the probability of getting three heads?
3.2 What is the probability of getting, at most, one tail

Question 4
The probability that a learner chooses Maths is 0.6, the probability of choosing History is 0.3 and the probability of choosing neither is 0.2. First draw the Venn diagram to represent the given information then use the diagram to calculate the probability of:
4.1 Choosing both subjects
4.2 Choosing only one subject
HINT: Let the intersection of Math and History be \(x\)

Question 5
A box contains 3 blue sweets and 2 green sweets. A sweet is drawn at random and then replaced. Another sweet is then taken from the box and replaced. Calculate the probability of:
5.1 first drawing a blue sweet and then a green sweet
5.2 first drawing a green sweet and then a blue sweet
5.3 not drawing a blue sweet on the first or second draw

Question 6
Consider three consecutive cricket matches. What is the probability that the captain of your favourite team will win the toss:
6.1 all three times
6.2 only once
6.3 at least once

Question 7
A bag contains six red and four blue marbles. A marble is drawn randomly and not replaced. A second marble is drawn and not replaced. Calculate the probability that:
7.1 the first marble is red
7.2 both marbles are red
7.3 one marble is red and the other is blue

**Question 8**
The Mathematics test results of 50 learners were examined and the results were recorded in the following two-way table:

<table>
<thead>
<tr>
<th></th>
<th>Passed</th>
<th>Failed</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class A</td>
<td>20</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Class B</td>
<td>13</td>
<td>12</td>
<td>25</td>
</tr>
<tr>
<td>Total</td>
<td>33</td>
<td>17</td>
<td>50</td>
</tr>
</tbody>
</table>

If a learner is chosen at random from the group, determine the following:
8.1 \( P(\text{the learner passed}) \)
8.2 \( P(\text{learner is from class B and passed}) \)
8.3 \( P(\text{learner is from class A and failed}) \)

**Question 9**
Sixty people were interviewed on their views of violence on TV and the results were recorded in the following two-way table:

<table>
<thead>
<tr>
<th></th>
<th>For violence</th>
<th>Against violence</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 25</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Over 25</td>
<td>3</td>
<td>27</td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td>13</td>
<td>47</td>
<td>60</td>
</tr>
</tbody>
</table>

If a person is chosen at random, calculate the following:
9.1 \( P(\text{over 25}) \)
9.2 \( P(\text{for violence on TV}) \)
9.3 \( P(\text{over 25 and for violence}) \)
9.4 \( P(\text{over 25}) \times P(\text{for violence}) \)
9.5 Do you think that views of violence on TV are independent of age? Why or why not?
Task Answers

Question 1
1.1 \( P(D) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{1}{6} \)

1.2 \( P(\text{vowel}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{2}{6} = \frac{1}{3} \)

1.3 \( P(\text{not vowel}) = 1 - P(\text{vowel}) = 1 - \frac{1}{3} = \frac{2}{3} \)

Question 2
2.1 Independent
2.2 Neither (the events are dependent)
2.3 Mutually exclusive

Question 3
S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}
3.1 Independent events so \( P(H \cap H \cap H) = P(H) \cdot P(H) \cdot P(H) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} \).
3.2 A = at most one tail = one tail or no tails = \{HHH, HHT, HTH, THH\} = 4/8 = \frac{1}{2} \)

Question 4

4.1 Let \( x \) be \( \text{M} \cap \text{H} \):
\[
0.6 - x + x + 0.3 - x + 0.2 = 1
\]
\[
\therefore x = 0.1
\]
\[
\text{So } P(\text{M} \cap \text{H}) = 0.1
\]
4.2 \( P(\text{only one subject}) = P(\text{M only}) + P(\text{H only}) = (0.6 - 0.1) + (0.3 - 0.1) = 0.7 \)

Question 5

Events are independent as the sweets are replaced each time.
5.1 \( P(\text{B and G}) = P(B) \cdot P(G) = \frac{3}{5} \cdot \frac{2}{5} = \frac{6}{25} \)
5.2 \( P(\text{G and B}) = P(G) \cdot P(B) = \frac{2}{5} \cdot \frac{3}{5} = \frac{6}{25} \)
5.3 \( P(\text{B}' \text{ and B}') = [1 - P(B)] \cdot [1 - P(B)] = \left(1 - \frac{3}{5}\right) \cdot \left(1 - \frac{3}{5}\right) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} \)
\[
\text{OR } P(\text{B}' \text{ and B}') = P(G \text{ and G}) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} \]
Question 6
The captain has a 50% chance of winning or losing each toss, the tree diagram looks like this:

6.1 \( P(w \cap w \cap w) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} \)

6.2 \( P(\text{win once}) = P(W \cap L \cap L) \cup P(L \cap W \cap L) \cup P(L \cap L \cap W) = P(W) \cdot P(L) \cdot P(L) + P(L) \cdot P(W) \cdot P(L) + P(L) \cdot P(L) \cdot P(W) \)
\(= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8} \)

6.3 \( P(\text{at least one win}) = P(\text{one or more wins}) = 1 - P(\text{no wins}) = 1 - P(L \cap L \cap L) = 1 - P(L) \cdot P(L) \cdot P(L) = 1 - \frac{1}{8} = \frac{7}{8} \)

Question 7

7.1 \( P(\text{first red}) = \frac{6}{10} = \frac{3}{5} \)

7.2 \( P(B \text{ and } B) = P(B) \cdot P(B) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90} = \frac{2}{15} \)

7.3 \( P(R \text{ and } B) \text{ or } P(B \text{ and } R) = P(R \cap B) + P(B \cap R) = \frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{5}{9} = \frac{48}{90} = \frac{8}{15} \)

Question 8

8.1 \( P(\text{learner passed}) = \frac{33}{50} = 0.66 \)

8.2 \( P(\text{learner is from class B and passed}) = \frac{13}{50} = 0.26 \)

8.3 \( P(\text{learner is from class A and failed}) = \frac{5}{50} = 0.1 \)
Question 9

9.1 \( P(\text{over 25}) = \frac{30}{60} = 0.5 \)

9.2 \( P(\text{for violence on TV}) = \frac{13}{60} \)

9.3 \( P(\text{over 25 and for violence}) = \frac{3}{60} = 0.05 \)

9.4 \( P(\text{over 25}) \times P(\text{for violence}) = \frac{1}{2} \times \frac{13}{60} = \frac{13}{120} \)

9.5 The opinions of violence on TV are dependent on age because:

\[ P(\text{over 25 and for violence}) = \frac{3}{60} = 0.05 \]

and \( P(\text{over 25}) \times P(\text{for violence}) = \frac{1}{2} \times \frac{13}{60} = \frac{13}{120} \)

\( P(\text{over 25 and for violence}) \neq P(\text{over 25}) \times P(\text{for violence}) \)

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