A Guide to Functions and Inverses

Teaching Approach

Functions and Inverses is covered in the first term of grade twelve in a period of about three weeks. Inverses of linear, quadratic and exponential functions have been dealt with. The series also cover the transformations.

The videos included in the Grade 12 Functions and Inverses do not have to be watched in any order. Summaries of the skills and contexts of each videos have been included in this document, allowing you to find something appropriate, quickly and easily.

Each video is short enough that it will fit into a lesson with time to discuss the content and some related work. You will find a selection of tasks covering the required skills in the task video. These tasks have not been linked to the videos so that they can be used without viewing them. The videos move from simple to complex.

When teaching functions it is always important to do some integration with aspects that have been covered before like you geometric transformation so that the work would make mathematical sense to the learners. This is a practical topic and learners are to do. The skills of learners will improve if they practice in different contexts.

Learners should be given time to explore and work with functions. Verbalisation of what they do or about to do will help in learner understanding.
Video Summaries

Some videos have a ‘PAUSE’ moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch or show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next day’s lesson; if desired, learners can be given specific questions to answer in preparation for the next day’s lesson

1. Defining a Function

We discuss the definition of a function. The distinction between a function and a relation is given. The test for a function is given. Mapping is used to show that some relations are not functions.

2. Vertical Translations

We discuss how we translate functions vertically. The upward and downward movements are explained. We also discuss the notation so that translations can be identified with ease.

3. Horizontal Translations

This video seeks to explain fully the movement horizontally of the functions. The notation is explained so that horizontal movements can be identified easily. The sign in the determines the direction of your movement.

4. Reflections of Functions

We have focus on the reflection on the axes in this video since the other types of transformations have been fully covered in other videos.

5. How to Stretch a Function

In this video, we show and explain how functions are stretched. The notation that gives us the stretch is given.

6. Discovering Inverse Functions

We define an inverse of a function. We discuss how we get the equation of an inverse given the equation of the original function. We integrate inverses with reflection in the line y=x. We give reasons why logarithms are used for inverses of exponential functions.
7. Inverse of a Linear Function
The method of getting the equation of an inverse of a linear function is discussed. It is also given that the gradient would remain the same but the y-intercept would most probable change.

8. Inverse of an Exponential Function
We discuss why we use the logs in the inverse of an exponential function. The asymptotes are fully explained. The use of the reflection line y=x is explored and expounded on. The log function is covered in this video.

9. Inverse of a Quadratic Function
The equation of the inverse of a quadratic function is discussed. The reason why the inverse of a quadratic function is not a function is given and tested. The use of the turning point to restrict the domain so that the resultant inverse can be a function is given.

Resource Material
Resource materials are a list of links available to teachers and learners to enhance their experience of the subject matter. They are not necessarily CAPS aligned and need to be used with discretion.

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td></td>
<td><a href="http://reference.wolfram.com/mathematica/tutorial/DefiningFunctions.html">http://reference.wolfram.com/mathematica/tutorial/DefiningFunctions.html</a></td>
<td>Worked examples (Functions tutorial)</td>
</tr>
<tr>
<td></td>
<td><a href="http://www.mathsisfun.com/sets/function.html">http://www.mathsisfun.com/sets/function.html</a></td>
<td>Worksheet on functions</td>
</tr>
<tr>
<td>2. Vertical Translations</td>
<td><a href="http://www.dummies.com/how-to/content/how-to-translate-a-functions-graph.html">http://www.dummies.com/how-to/content/how-to-translate-a-functions-graph.html</a></td>
<td>Defines vertical translation</td>
</tr>
<tr>
<td></td>
<td><a href="http://mathmaine.wordpress.com/2010/05/27/function-translations/">http://mathmaine.wordpress.com/2010/05/27/function-translations/</a></td>
<td>Notes on translations</td>
</tr>
<tr>
<td></td>
<td><a href="http://staff.argyll.epsb.ca/jreed/math30p/transformations/translations.htm">http://staff.argyll.epsb.ca/jreed/math30p/transformations/translations.htm</a></td>
<td>Worksheet on vertical shift</td>
</tr>
<tr>
<td>3. Horizontal Translations</td>
<td><a href="http://www.dummies.com/how-to/content/how-to-translate-a-functions-graph.html">http://www.dummies.com/how-to/content/how-to-translate-a-functions-graph.html</a></td>
<td>Defines vertical translation</td>
</tr>
<tr>
<td></td>
<td><a href="http://www.analyzemath.com">www.analyzemath.com</a></td>
<td>Interactive vertical shifting</td>
</tr>
<tr>
<td></td>
<td><a href="http://mathmaine.wordpress.com/2010/05/27/function-translations/">http://mathmaine.wordpress.com/2010/05/27/function-translations/</a></td>
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<td>Worksheet on vertical shift</td>
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<tr>
<td></td>
<td><a href="http://home.windstream.net/okreb/page43.html">http://home.windstream.net/okreb/page43.html</a></td>
<td>Defining a stretch</td>
</tr>
<tr>
<td></td>
<td><a href="http://www.themathpage.com/aprecalc/reflections.htm">http://www.themathpage.com/aprecalc/reflections.htm</a></td>
<td>Stretching of graphs</td>
</tr>
<tr>
<td>5. How to Stretch a Function</td>
<td><a href="http://www.sparknotes.com/math/algebra2/operationsonfunctions/section2.rhtm">http://www.sparknotes.com/math/algebra2/operationsonfunctions/section2.rhtm</a></td>
<td>Interactive question and answer on application of functions</td>
</tr>
<tr>
<td></td>
<td><a href="http://www.youtube.com/watch%3Fv%3DUG_zXtVBsh0">http://www.youtube.com/watch%3Fv%3DUG_zXtVBsh0</a></td>
<td>Teacher’s guide (transformation applications)</td>
</tr>
<tr>
<td></td>
<td><a href="http://staff.argyll.epsb.ca/ireed/math30p/transformations/stretches.htm">http://staff.argyll.epsb.ca/ireed/math30p/transformations/stretches.htm</a></td>
<td>Applications of graph transformation:</td>
</tr>
<tr>
<td>6. Discovering Inverse Functions</td>
<td><a href="http://www.mathsisfun.com/sets/function-inverse.html">http://www.mathsisfun.com/sets/function-inverse.html</a></td>
<td>Worked examples</td>
</tr>
<tr>
<td>7. Inverse of a Linear Function</td>
<td><a href="http://www.augustatech.edu/math/molik/InverseLinear.pdf">http://www.augustatech.edu/math/molik/InverseLinear.pdf</a></td>
<td>The procedure for finding the inverse of a linear function</td>
</tr>
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<td></td>
<td><a href="http://www.classzone.com/eservices/home/pdf/teacher/LA207DBD.pdf">http://www.classzone.com/eservices/home/pdf/teacher/LA207DBD.pdf</a></td>
<td>The equation of the inverse of a linear function</td>
</tr>
<tr>
<td>8. Inverse of an Exponential Function</td>
<td><a href="http://www.math.uh.edu/~ajajoo/MATH1310/LectureNotes/1310_Notes_5o3_fill2.PDF">http://www.math.uh.edu/~ajajoo/MATH1310/LectureNotes/1310_Notes_5o3_fill2.PDF</a></td>
<td>The graph of the logarithm and exponential function</td>
</tr>
<tr>
<td></td>
<td><a href="http://math.stackexchange.com/questions/327896/how-to-find-the-inverse-of-this-exponential">http://math.stackexchange.com/questions/327896/how-to-find-the-inverse-of-this-exponential</a>...</td>
<td>How to inverse exponential functions</td>
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<td>Video of exponential function</td>
</tr>
<tr>
<td>9. Inverse of a Quadratic Function</td>
<td><a href="http://www.wikihow.com/Find-the-Inverse-of-a-Quadratic-Function">http://www.wikihow.com/Find-the-Inverse-of-a-Quadratic-Function</a></td>
<td>To determine inverse of a quadratic function</td>
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<td></td>
<td><a href="http://www.algebra.com/algebra/homework/quadratic/Quadratic_Equations.faq.question.202334.h.h">http://www.algebra.com/algebra/homework/quadratic/Quadratic_Equations.faq.question.202334.h.h</a></td>
<td>Worksheet and answers</td>
</tr>
<tr>
<td></td>
<td><a href="http://www.eowyn.org/teaching/AdvAlgebra/Functions_03.pdf">http://www.eowyn.org/teaching/AdvAlgebra/Functions_03.pdf</a></td>
<td>Lesson plans and worksheet</td>
</tr>
<tr>
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<td>Worksheets on transformation of exponential function</td>
</tr>
<tr>
<td></td>
<td><a href="http://www.purplemath.com/modules/invrscfc2.htm">http://www.purplemath.com/modules/invrscfc2.htm</a></td>
<td>Properties of exponential functions</td>
</tr>
<tr>
<td></td>
<td><a href="http://www.analyzemath.com/inversefunction/inverse_quadratic.html">http://www.analyzemath.com/inversefunction/inverse_quadratic.html</a></td>
<td>Teacher school worksheets</td>
</tr>
</tbody>
</table>
Task

Question 1
Given \( f(x) = 9^x \) and \( g(x) = 2(x - 3)^2 - 18 \)
1.1 Sketch the graphs of \( f \) and \( g \) on the same set of axes. Indicate Intercepts and any turning points
1.2 Find \( f^{-1} \) in the form \( y = \ldots \)
1.3 Sketch the graph of \( f^{-1}(x - 2) \) on the same set of axes indicate any two points
1.4 For which values of \( x \) is the graph of \( f^{-1}(x - 2) < 3 \)
1.5 Describe the transformation that maps \( g(x) \) onto \( h(x) = -2(x - 3)^2 + 5 \)
1.6 Show algebraically, that \( f(x + \frac{1}{2}) = 3f(x) \)

Question 2
The graph of \( m(x) = b^x \) is sketched. \( A(-1; \frac{1}{4}) \) is a point on the graph of \( m \).

2.1 Write down the co-ordinates of the \( y \) – intercept. Give a reason for your answer
2.2 Calculate the value of \( b \)
2.3 Write down the equation of the inverse function, \( m^{-1} \) in the form \( y = \ldots \)
2.4 What is the domain and the range of \( m \)?
2.5 Write down the equation of the asymptote of \( l(x) \) if \( l(x) = m(x) + 2 \)

Question 3
Sketched below are the graphs of \( f(x) = 3^x \) and \( g(x) = -(x - 1)^2 + b \), where \( b \) is a constant. The graphs of \( f \) and \( g \) intersect at \( C \). \( D \) is the turning point of \( g \).

3.1 Show that \( b = 3 \)
3.2 Write down the coordinates of the turning point of \( g \).
3.3 Write down the equation of \( f^{-1}(x) \) in the form of \( y = \ldots \)
3.4 Write down the domain and the range of \( f^{-1} \)
3.5 Sketch the graph of \( g^{-1} \)
3.6 How can the domain of \( g \) be restricted so that \( g^{-1} \) will be a function?
3.7 Write down the equation of \( h \) if \( h(x) = g(x + 1) - 2 \).
3.8 Explain why the maximum value of \( 3^{3-(x-1)^2} \) is 27

**Question 4**

Sketched below are the graphs of \( f(x) = a(x - p)^2 + q; \ g(x) = mx + c \) and \( h(x) = \frac{6}{x-2} + 1 \).

Points \((-1;10)\) and \((1;2)\) are on \( g \). The intercepts of \( f \) are at \((-1;0)\) and \((5;0)\) and \((0;-10)\).

4.1 Find the \( x \) and \( y \)-intercepts of \( h \)
4.2 Find the equation of \( f \) and \( g \)
4.3 Write the equations of the asymptotes of \( h \)
4.4 If \( m(x) = g(-x) \), write down the equation of \( m \).
4.5 Given \( k(x) = -f(x) \); write down the turning points of \( k \)
4.6 Sketch \( l \) if \( l(x) = -3g(x) \)
4.7 Determine the values of \( x \) for which \( g(x) - f(x) \geq 0 \)
Task Answers

Question 1

1.1

1.2 \( f(x) = 9^x \)
   
   \( x = 9^y; \)
   
   \( y = \log_9 x \)

1.3

1.4 \( \ln = 9^{x-2} \); when \( x = 3; \ y = 9 \);

\( \therefore \) in the inverse of \( y = 9^{x-2} \), when \( y = 3 \) then \( x = 9 \)

and \( x = 0 \) is the asymptote of \( y = \log_9 x - 2 \)

we can conclude that for \( f(x - 2) < 3; \quad 0 < x < 9 \)

1.5 It’s a reflection in the x-axis followed by a translation of 13 units down.

1.6 \( f(x + \frac{1}{2}) = 9^{x + \frac{1}{2}} \)

\[ = 9^x \cdot 9^{\frac{1}{2}} \]

\[ = 9^x \cdot 3 \]

\( \therefore \) \( 3f(x) \)

Question 2

2.1 The coordinates for the y-axis are \((0;1)\) because \( b^0 = 1 \)
2.2 \( \frac{1}{4} = b^{-1} \)

\[ \frac{1}{4} = \frac{1}{b} \]

Since the numerators are equal the denominators must be equal as well.
Therefore \( b = 4 \).

2.3 \( m(x) = 4^{x} \)
\[ x = 4^{y} \]
\[ y = \log_{4}x \]

2.4 \( x \in R, \ y > 0 \)

2.5 \( y = 2 \)

**Question 3**

3.1 At the point of intersection of the two graphs \( x=1 \) and \( 3^{1} \) therefore the value of \( b \) is 3.

3.2 The turning points are (1;3)

3.3 \( y = \log_{3}x \)

3.4 \( x > 0 \ and \ y \in R \)

3.5

3.6 \( x < 1 \ or \ x > 1 \)

3.7 \( h(x) = g(x + 1) - 2 \)
\[ = -(x + 1 - 1)^{2} + 3 - 2 \]
\[ = -(x^{2} + 1) \]

3.8 We get the maximum when \( (x-1)^{2} \) is least. \( (x-1)^{2} \) is least when \( x \) is equal to 2. Because that is the only time we get a zero, otherwise all other values will give us a positive answer.
Question 4

4.1 For the \(x\)-intercept

\[0 = \frac{6}{x-2} + 1\]
\[0 = 6 + (x - 2)\]
\[6 + x - 2 = 0\]
\[x = -4\]

For the \(y\)-intercept

\[y = \frac{6}{x-2} + 1\]
\[y = -3 + 1 ; y = -2\]

4.2 \[m = \frac{10-2}{-1-1}\]
\[m = -4\]
\[y - 2 = -4(x - 1)\]
\[y - 2 = -4x + 4\]
\[y = -4x + 6\]
\[g(x) = -4x + 6\]

For \(f\):
\[y = a(x - p)(x - q)\]
\[y = a(x + 1)(x - 5)\]
\[-10 = a(0 + 1)(0 - 5)\]
\[-10 = -5a\]
\[2 = a\]
\[y = 2(x + 1)(x - 5)\]
\[y = 2(x^2 - 4x - 5)\]
\[y = 2x^2 - 8x - 10\]
\[f(x) = 2x^2 - 8x - 10\]

4.3 \(x = 2\) and \(y = 1\)

4.4 \[m(x) = -4(-x) + 6\]
\[m(x) = 4x + 6\]

4.5 Turning point of \(k\)

\[k(x) = -[2x^2 - 8x - 10]\]
\[k(x) = -2x^2 + 8x + 10\]
\[k(x) = -2[x^2 - 4x - 5]\]
\[k(x) = 2[x^2 - 4x + 4 - 4 - 5]\]
\[k(x) = 2[(x - 2)^2 - 9]\]
\[k(x) = 2(x - 2)^2 - 18\]

Therefore the turning points of \(k(x)\) are (2; -18)
4.6 \( l(x) = -3(-4x + 6) \)
\( l(x) = 12x - 18 \)

4.7 \(-4x + 6 - (2x^2 - 8x - 10) \geq 0\)
\(-4x + 6 - 2x^2 + 8x + 10 \geq 0\)
\(-2x^2 + 4x + 16 \geq 0\)
\(2x^2 - 4x - 16 \leq 0\)
\(2(x - 4)(x + 2) \leq 0\)
\(-2 \leq x \leq 4\)

**Acknowledgements**

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