A Guide to Advanced Analytical Geometry

Teaching Approach

Before starting with the Grade 12 Advanced Analytical Geometry Series it is recommended that revision is done of all Grade 11 Analytical Geometry. Revise all analytical formulas used in Grade 11 and give the pupils a revision exercise to complete.

The pupils can also look at the Grade 10 and Grade 11 Analytical Geometry Series from Mindset Learn to revise the work taught in these grades.

Start the Grade 12 chapter by finding the equation of a circle with centre as origin. Then move on to finding the equation of a circle where the centre is not the origin. Here you will have to revise completing the square that was taught in Grade 11.

The last section in Grade 12 Advanced Analytical Geometry is finding the equation of a tangent to a circle. Revise the Grade 11 Euclidean geometry theorem, which states that the radius is perpendicular to the tangent and link that to what they learnt in Grade 11 about the product of the gradients of two perpendicular lines.

It is important to extend the pupils and work with them through more difficult examples to ensure that they are adequately prepared for examination type questions.

Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch of show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

1. **Circles with Centre on the Origin**

   In this video the equation of a circle with the origin as centre is found.

2. **Circles not Centred at the Origin**

   In this video we find the equation of a circle where the centre is not the origin. The equation of the circle is found either by having the radius and the centre co-ordinates given, or by the radius and a point on the circle given.

3. **Finding the Equation of a Tangent to a Circle**

   In this video the equation of the tangent to a circle is found.

Resource Material

Resource materials are a list of links available to teachers and learners to enhance their experience of the subject matter. They are not necessarily CAPS aligned and need to be used with discretion.

1. Circles with Centre on the Origin	http://www.mathsvillage.org/universitymaths/circles.htm	Notes on finding the equation of a circle with centre as origin
	http://www.youtube.com/watch?v=HjN9TTRrQiA	Notes and examples on finding the equation of a circle with centre the origin as well as centre not being the origin.
	http://www.artmathonline.com/files/AG26aa.pdf	A video on finding the equation of a circle with the centre as origin
2. Circles not Centred at the Origin	http://www.mathsvillage.org/universitymaths/circles.htm	Notes on finding the equation of a circle with centre the origin as well as centre not being the origin.
	http://www.purplemath.com/modules/sqrcircle.htm	Notes and examples on completing the square of circle equations.
	http://www.artmathonline.com/files/AG26aa.pdf	Notes and examples on finding the equation of a circle with centre the origin as well as centre not being the origin.
3. Finding the Equation of a Tangent to a Circle	http://www.youtube.com/watch?v=30qK9QE_E8A	A video on finding the equation of a tangent to a circle.
	http://www.youtube.com/watch?v=7Ufv0X5y0fc	This is a YouTube video that deals with a revision on calculus questions.
	http://academic.sun.ac.za/mathed/shoma/MATUNIT13_07.htm	Notes on finding the equation of a circle with centre the origin as well as centre not being the origin. As well as finding the equation of a tangent to a circle.

Task

Question 1

Determine the equation of the circle with centre A(3; -5) and passing through the point B(1; -1).

Question 2

Determine the radius of each of the following circles:

2.1 $x^2 + y^2 = 100$

2.2 $ax^2 + ay^2 = b$

Question 3

Determine the equation of the circle with centre as origin and a radius of $2\sqrt{7}$.

Question 4

Determine the equation of the circle with the origin as centre and passing through the point (-2; 3)

Question 5

A (-5; 12) is a point on a circle, centre the origin.

5.1 Determine the equation of the circle

5.2 Determine the co-ordinates of C if AC is the diameter of the circle.

5.3 Show that the point J(13; 0) lies on this circle.

Question 6

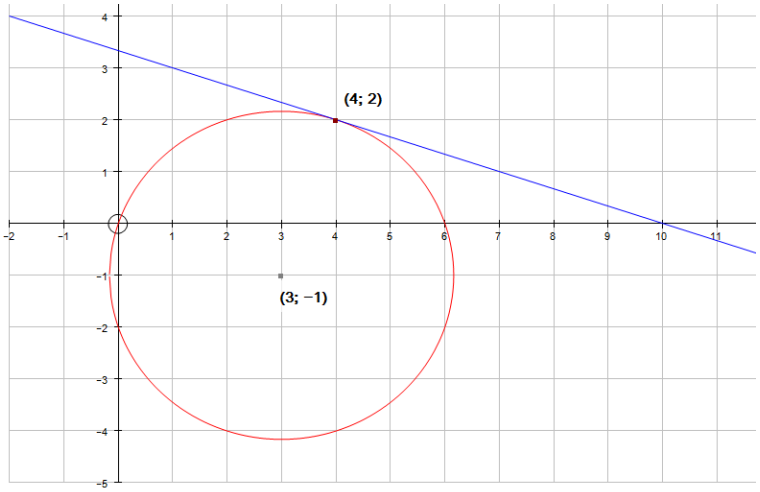
Determine the centre and the radius of the following circle, $(x-1)^2 + (y-3)^2 = 16$

Question 7

Determine the centre and the radius of the following circle, $x^2 + y^2 - 8x - 2y - 20 = 0$

Question 8

Find the equation of the tangent to the circle $x^2 - 6x + y^2 + 2y = 0$



Question 9

A circle with equation $x^2 + y^2 - 4x + 2y + k = 0$ passes through the point (1; 3).

Determine

9.1 the centre of the circle

9.2 the value of k

9.3 the radius of the circle in surd form

9.4 the equation of the tangent to the circle at (1; 3).

Question 10

Prove that the circles $x^2 + y^2 = 16$ and $(x - 4)^2 + (y + 3)^2 = 1$touch each other.

Task Answers

Question 1

$(x-3)^2+(y+5)^2=r^2$ through (1;-1)

$\therefore (1-3)^2+(-1+5)^2=r^2$

$\therefore 20=r^2$

$\therefore (x-3)^2+(y+5)^2=20$

$\therefore x^2-6x+y^2+10y+14=0$

Question 2

2.1 $x^2+y^2=100$

$\therefore r^2=100$

$\therefore r=\pm\sqrt{100}$

$\therefore r=\pm 10$

$\therefore r=10$

The length of the radius can't be negative so the final answer should be positive.

2.2 $ax^2+ay^2=b$

$a(x^2+y^2)=b$

$x^2+y^2=\frac{b}{a}$

$\therefore r^2=\frac{b}{a}$

$\therefore r=\pm\sqrt{\frac{b}{a}}$

$\therefore r=\sqrt{\frac{b}{a}}$ The length of the radius can't be negative so the final answer should be positive

Question 3

$x^2+y^2=(2\sqrt{7})^2$

Question 4

$x^2+y^2=r^2$

$(-2)^2+(3)^2=r^2$

$\therefore r^2=13$

$x^2+y^2=13$

Question 5

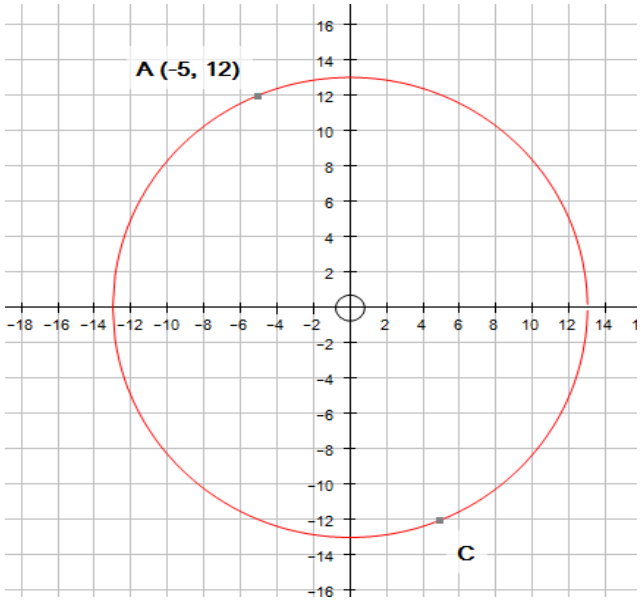
5.1 $x^2 + y^2 = r^2$

$(-5)^2 + (12)^2 = r^2$

$\therefore r^2 = 169$

$x^2 + y^2 = 169$

5.2



$\therefore C(5; -12)$

5.3 $x^2 + y^2 = 169$

Question 6

$(x-1)^2 + (y-3)^2 = 16$

Centre (1; 3)

$r = \pm\sqrt{16}$

$\therefore r = 4$

Question 7

$x^2 + y^2 - 8x - 2y - 20 = 0$

$x^2 - 8x + y^2 - 2y = 20$

$x^2 - 8x + (-4)^2 + y^2 - 2y + (-1)^2 = 20 + 16 + 1$

$\therefore (x-4)^2 + (y-1)^2 = 37$

$\therefore$ Centre (4; 1) and radius $\sqrt{37}$

Question 8

$x^2 - 6x + y^2 + 2y = 0$

$x^2 - 6x + (-3)^2 + y^2 + 2y + (1)^2 = 0 + 9 + 1$

$\therefore (x-3)^2 + (y+1)^2 = 10$

Gradient of the radius $\frac{2-(-1)}{4-3} = 3$

$\therefore$ Gradient of the tangent = $-\frac{1}{3}$

$\therefore y = -\frac{1}{3}x + c$

$\therefore 2 = -\frac{1}{3}(4) + c$

$\therefore \frac{10}{3} = c$

Equation of the tangent $y = -\frac{1}{3}x + \frac{10}{3}$

Question 9

9.1 $x^2 + y^2 - 4x + 2y + k = 0$

$x^2 - 4x + y^2 + 2y = -k$

$x^2 - 4x + 4 + y^2 + 2y + 1 = -k + 4 + 1$

$\therefore (x-2)^2 + (y+1)^2 = -k + 5$

$\therefore$ Centre co-ordinates(2; -1)

9.2 $(1-2)^2 + (3+1)^2 = -k + 5$

$(-1)^2 + (4)^2 = -k + 5$

$17 = -k + 5$

$\therefore k = -12$

9.3 $(x-2)^2 + (y+1)^2 = -k + 5$

$(x-2)^2 + (y+1)^2 = -(-12) + 5$

$(x-2)^2 + (y+1)^2 = 16$

$\therefore$ Radius = $\sqrt{17}$

9.4 (2; -1) and (1; 3)

Gradient of radius: $\frac{3-(-1)}{1-2} = \frac{4}{-1} = -4$ $\therefore$ Gradient of tangent = $\frac{1}{4}$

$$\therefore y = \frac{1}{4}x + c$$

$$\therefore 3 = \frac{1}{4}(1) + c$$

$$\therefore \frac{1}{4} = c$$

$$\therefore y = \frac{1}{4}x + \frac{1}{4}$$

Question 10

Circle $x^2 + y^2 = 16$ has a centre (0; 0)

Circle $(x-4)^2 + (y+3)^2 = 1$ has a centre (4; -3)

Distance from (0;0) to (4; -3) $= \sqrt{(4)^2 + (-3)^2}$

$= \sqrt{25}$

$= 5$

Radius of circle $x^2 + y^2 = 16$ is 4

Radius of circle $(x-4)^2 + (y+3)^2 = 1$ is 1

The distance between the centres of the circles is 4 + 1 =5
Circles will touch when the distance between their centres is equal to the sum of their radii.

Acknowledgements

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