A Guide to Graphs and Equations of Motion

Teaching Approach

In this series we explain the difference between instantaneous and average velocity. We use the action of a ticker tape to show how the graphs of motion are formed, and explain how these can be used to calculate other unknowns. This is done for both constant velocity and constant acceleration. The difference between positive and negative acceleration is highlighted in the lessons. The origin and development of the equations of motion are shown. Finally, we show how to work step by step through an equation of motion problem to solve it. The concept that negative velocity can mean both slowing down in the same direction, or speeding up in the opposite direction might be a concept that learners might struggle with. The best way to help learners understand this is to point out that the further the graph is away from the x-axis on a velocity versus time graph, the faster the object is going, no matter which side of the x-axis the graph is on.

Using ticker timers or even someone on a skateboard with a leaky bucket of a washable but colourful substance is a good way for the learners to see that when you have constant velocity the spaces between the marks are consistent, and if there is acceleration the gaps between the marks becomes larger.

Finally, the videos can either be used independently for the learners to revise with after learning the content at school, or as part of the lessons. If it is possible, the use of multimedia to explain and reinforce concepts helps the learners understand and grasp new concepts better.

The task video has been prepared in such a way as it could be used as either an exercise tool, or it can be used as a complete test for the learners, or as a way for the learners to test their knowledge independently.
Video Summaries

Some videos have a ‘PAUSE’ moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch or show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next day’s lesson; if desired, learners can be given specific questions to answer in preparation for the next day’s lesson

1. Instantaneous Velocity

In this lesson we revise the fact that velocity equals the change in displacement over time. It is shown that if there is acceleration, the instantaneous velocity is different from the average velocity.

2. Graphs of Motion for Constant Velocity

In this lesson we see how the motion of a ticker tape and the measurements made, can be used to plot a displacement versus time graph.

3. Constant Acceleration and Graphs of Motion

In this lesson we investigate constant acceleration. We show how the gradient of the velocity versus time graph can be used to calculate the acceleration at a point.

4. Revision of Graphs of Motion

In this lesson we revise all the graphs of motion for both constant velocity and constant acceleration.

5. Graphs of Motion Practice

In this lesson we use knowledge we have gained and do examples to interpret graphs of motion. We apply the techniques discussed in the previous lessons, using the graphs to calculate displacement, velocity and acceleration.

6. Equations of Motion

In this lesson we introduce equations of motion. We show how the graphs of motion and the basic definition of velocity can be used to develop the four equations of motion. We also give a step by step guide on how to do an equation of motion problem.
## Resource Material

<table>
<thead>
<tr>
<th></th>
<th>Instantaneous Velocity</th>
<th>Calculating Instantaneous velocity from the average velocities</th>
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<td></td>
<td><a href="http://www.youtube.com/watch?v=pU9DuRA0g1Q">http://www.youtube.com/watch?v=pU9DuRA0g1Q</a></td>
<td>This page defines instantaneous velocity.</td>
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<td><a href="http://www.ask.com/question/what-is-instantaneous-velocity">http://www.ask.com/question/what-is-instantaneous-velocity</a></td>
<td>Here you will find instructions on how to calculate instantaneous velocity.</td>
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<th>Constant Velocity and Graphs of Motion</th>
<th>This page describes motion with velocity vs. time graphs</th>
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<td>A resource on motion with constant velocity.</td>
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<td><a href="http://farside.ph.utexas.edu/teaching/301/lectures/node17.html">http://farside.ph.utexas.edu/teaching/301/lectures/node17.html</a></td>
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<tr>
<th></th>
<th>Constant Acceleration and Graphs of Motion</th>
<th>In this video, we look at motion graphs for an example of a ball.</th>
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<td><a href="http://www.youtube.com/watch?v=dGDioMWJs-Q">http://www.youtube.com/watch?v=dGDioMWJs-Q</a></td>
<td>A video on straight line motion with constant acceleration</td>
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<td>Learner resource on motion graphs</td>
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<th>Practicing Graphs of Motion</th>
<th>Practice problems.</th>
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6. **Equations of Motion**

<table>
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<th>Resource</th>
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<td>A resource on kinematic equations and problem solving.</td>
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Task

Question 1
The graph below describes the straight line motion of a car over a period of 60s.

1.1 Use the graph to calculate the magnitude of the acceleration during the first 20 s
1.2 Use the graph to calculate the displacement obtained during the 60 s.
1.3 Draw a neat sketch graph of acceleration versus time for the full 60 s. Label the axes and show appropriate time and acceleration values.
1.4 Draw a neat sketch graph of displacement versus time for the full 60 s. Just show the displacement values at 0, 20, 50 and 60 s.

Question 2
An athlete competes in the Commonwealth Games. His average speed is 10.32 m.s$^{-1}$. What is his average speed in kilometres per hour?

Question 3
This a velocity versus time graph for an athlete’s motion during a 100 m race. The race is divided into 3 stages: A, B and C.

3.1 Use the graph to calculate the athlete’s acceleration during the first 3.2 s.
3.2 Use one of the equations of motion to calculate the distance that the athlete ran in Stage C.
3.3 Draw the corresponding acceleration versus time graph for the athlete’s motion.

Question 4
The speed limit near a school is 30 km·h$^{-1}$ but Mr Smith thinks he can ignore it. He drives at 60 km·h$^{-1}$, AND he is talking on his cell phone. Suddenly a child runs into the road after a ball. The maximum rate that Mr Smith can slow down at is 8 m·s$^{-2}$.
4.1 Calculate the distance away from the car the child needs to be, in order not to be hurt.

4.2 We are told that the time it takes for an alert person to react is about 1.5 s, while if someone tired, drunk or distracted by a cell phone the time it takes for a person to apply brakes can be three seconds.

Complete the table below:

<table>
<thead>
<tr>
<th>Driver’s State</th>
<th>Reaction Time</th>
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<tbody>
<tr>
<td>Alert</td>
<td></td>
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<tr>
<td>Distracted or intoxicated.</td>
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</table>

4.3 Use information above to calculate how far away Mr Smith must be when he first sees the child (and applies brakes) in order to stop in time.

4.4 Why do you think it is bad for us to drink and drive and use cell phones when we drive?

**Question 5**

An aeroplane starts from rest and accelerates at 12 m.s⁻² for 8 s before it takes off.

5.1 Calculate the take-off speed in kilometres per hour.

5.2 Calculate the minimum length of runway required.


**Task Answers**

**Question 1**

1.1 Acceleration = $\frac{\Delta v}{\Delta t}$

$$= \frac{30-0}{20-0}$$

= 1.5 m$\cdot$s$^{-2}$

1.2 Displacement = area under the graph

$$= \frac{1}{2}(20)(30) + (50 - 20)(30) + \frac{1}{2}(60 - 50)(30)$$

= 300 + 900 + 150

= 1350 m

1.3

1.4

**Question 2**

Speed = $\frac{10.32}{1000} \times 3600 = 37.15$ km$\cdot$h$^{-1}$

**Question 3**

3.1 Acceleration = $\frac{\Delta v}{\Delta t}$

$$= \frac{12-0}{3.2-0}$$

= 3.75 m$\cdot$s$^{-2}$

3.2 $v_i = 12$

$v_f = 10$

$\Delta t = 2$
3.3
\[ \Delta x = \frac{(v_f - v_i)}{2} \Delta t \]
\[ = \left( \frac{10 - 12}{2} \right) \times 2 \]
\[ = -1 \times 2 \]
\[ = -2 \text{ m} \]
Therefore the distance run was 2 metres.

Question 4

4.1 First we need to convert Mr Smith's speed from \(\text{km} \cdot \text{h}^{-1}\) to \(\text{m} \cdot \text{s}^{-1}\):

\[ \text{Speed} = 60 \times \frac{1000}{3600} = 16.67 \text{m} \cdot \text{s}^{-1} \]

\[ v_i = 16.67 \]

\[ v_f = 0 \]

\[ a = -8 \]

\[ \Delta x = ? \]

\[ v_f^2 = v_i^2 + 2a\Delta x \]

\[ \Delta x = \frac{v_f^2 - v_i^2}{2a} \]

\[ = \frac{0 - (16.67)^2}{2(-8)} \]

\[ = 17.37 \text{ m} \]

4.2

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<td>3</td>
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4.3 The driver travels at a constant speed of 16.67 \(\text{m} \cdot \text{s}^{-1}\) until he reacts. If he is distracted he will take 3 seconds (on average) to respond which means that he will have travelled:

\[ \Delta x = 16.67 \times 3 \]

\[ = 50.01 \text{ m} \]

Therefore he needs to be at least 50.01 + 17.37 = 67.38 m away from the child in order to stop in time.

4.4 Being intoxicated or using cell phones greatly increases our reaction time, resulting in a greater chance of us having an accident.

Question 5

5.1

\[ v_i = 0 \]

\[ v_f = ? \]

\[ a = 12 \]

\[ t = 8 \]

\[ v_f = v_i + a\Delta t \]

\[ = 0 + 12(8) \]
\[ v_i = 96 \text{ m} \cdot \text{s}^{-1} \]
\[ \text{Speed} = \frac{96}{1000} \times 3600 = 345.6 \text{ km} \cdot \text{h}^{-1} \]

5.2 \( v_i = 0 \)
- \( a = 12 \)
- \( t = 8 \)
- \( \Delta x = ? \)

\[ \Delta x = v_i \Delta t + \frac{1}{2} a (\Delta t)^2 \]
\[ = 0 + \frac{1}{2}(12)(8^2) \]
\[ \Delta x = 384 \text{ m} \]
Acknowledgements

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